2019/20-ME
MATH CP
PAPER 1

HOK YAU CLUB

HONG KONG MOCK EXAMINATION 2019/20

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

9:00 am – 11:15 am (2¼ hours)

This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1 and 3.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Question No.	Marks	Marks
1-2		
3-4		
5-6		
7		
8		
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11		
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Total		

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SECTION A (1) (35 marks)

1. Make m the subject of the formula $\frac{2(m+2n)}{3} = 3m - 2$. (3 marks)

2. Simplify $\frac{3}{3x-5} + \frac{2}{3-2x}$. (3 marks)

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Page total

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3. Simplify $\frac{(a^{-3})^2}{(a^8b^{-2})^{-3}}$ and express your answer with positive indices. (3 marks)

4. Factorize

(a) $3a^3 + a^2b - 2ab^2$,

(b) $3a^3 + a^2b - 2ab^2 - 6a + 4b$.

(4 marks)

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5. The marked price of a book is \$168 . The book is now sold at a discount of 25% on its marked price.

(a) Find the selling price of the book.

(b) It is given that the cost of the book is \$90 , find the percentage profit of the book .

(4 marks)

6. (a) Solve the inequality $4(x+2) > \frac{11x-5}{5}$.

(b) Find the least integer satisfying both inequalities $4(x+2) > \frac{11x-5}{5}$ and $5+2x \geq -6$.

(4 marks)

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7. In a polar coordinate system, O is the pole. The polar coordinates of the point A are $(24, 27°)$. A is rotated clockwise about the pole O through $60°$ to point A'. It is given that the straight line L bisects $\angle AOA'$.

(a) Write down the polar coordinates of A'.

(b) Describe the geometric relationship between L and the line segment AA'.

(b) If B is a point lying on L such that $OABA'$ is a rhombus, find the polar coordinates of B.

(4 marks)

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8. In Figure 1 , AC is the diameter of the circle , the chords BD and AC intersect at the point E .

It is given that $3\overset{\frown}{CD} = 4\overset{\frown}{AB}$ and $\angle BEC = 96°$.

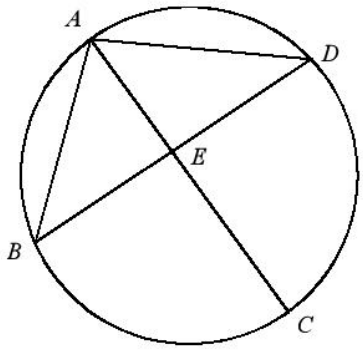

Figure 1

(a) Find $\angle ADB$.

(b) Find $\angle ABD$.

(5 marks)

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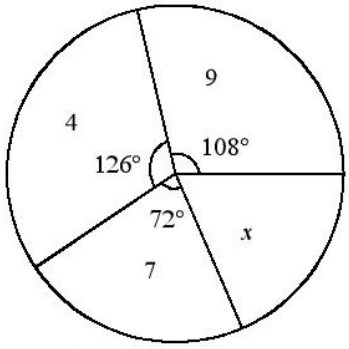
9. The pie chart below shows the distribution of the numbers of books read by a group of students in a month, where $4 < x < 9$. It is given that the mean of the distribution is 6.25 .



The distribution of the numbers of books read by this group of students in a month

(a) Write down the interquartile range of the distribution.

(b) Find x .

(c) If a student is randomly selected from the group, find the probability that the selected student reads less than 7 books.

(5 marks)

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SECTION A (2) (35 marks)

10. Let $h(x)$ partly varies as x and partly varies as x^2. Suppose that $h(-2) = 8$ and $h(3) = 33$.

(a) Find $h(x)$. (3 marks)

(b) Solve the equation $h(2x) = 16$. (3 marks)

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11. In Figure 2 , ABC is a triangle. D and E are points lying on BC and AC respectively such that $\angle ABD = \angle ADE = \angle DCE$.

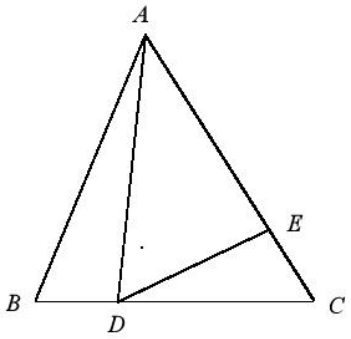

Figure 2

(a) Prove that $\Delta ABD \sim \Delta DCE$. (3 marks)

(b) Suppose that $AE = 14$, $BD = 6$, $AD = 6\sqrt{7}$ and $DE = 4\sqrt{7}$. Is ABC an equilateral triangle?

Explain your answer. (3 marks)

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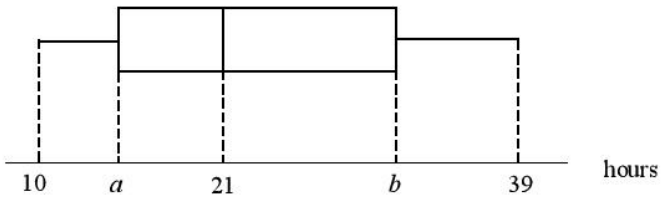
12. The stem-and-leaf diagram below shows the numbers of online hours spent by 20 students in a certain week:

Stem (tens)	Leaf (units)
1	0 2 2 4 5 5 7 8 9 9
2	k 4 6 7 8 9
3	2 5 6 9

where $0 \le k \le 4$, and k is an integer.

The box-and-whisker diagram below shows the distribution of the numbers of online hours:



(a) Find a, b and k. (3 marks)

(b) It is given that the mean of the numbers of online hours spent by 4 students is the same as the mean of the numbers of online hours spent by the original 20 students. It is found that two of these four data are 23 and 24. Is it possible that the median of the numbers of online hours spent by these 24 students is the same as the median of the numbers of online hours spent by the original 20 students? Explain your answer.

(4 marks)

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13. Let $p(x)$ be a cubic polynomial. It is given that $p(x)$ is divisible by $x^2 - x - 6$. When $p(x)$ is divided by $x+1$, the remainder is 4. When $p(x)$ is divided by $x-2$, the remainder is -20.

(a) Find $p(x)$. (3 marks)

(b) Let $q(x) = (x-3)(2x+k)$, where k is a constant. Find the range of values of k such that the equation $p(x) = q(x)$ has three distinct real roots. (5 marks)

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14. An hourglass is formed by connecting the vertices of two identical right circular conical container of radius 3 cm . It is given that the height of the hourglass is 12 cm. The hourglass is held vertically at the beginning and all the sand is located in the upper part of the inverted circular conical container. The surface of the sand remains horizontal and the height of the sand is h cm .

(a) Find the volume of the sand in terms of π. (3 marks)

(b) The sand now flows into the lower part of the circular conical container from the upper part of the inverted circular conical container at a constant speed of $\frac{4\pi}{45}$ cm^3 per second, and all the sand drains away in just 1 minute.

(i) Find h.

(ii) When all the sand flows into the lower part of the circular conical container, the surface of the sand remains horizontal. A student claims that the height of the sand is less than 1 cm . Do you agree? Explain your answer.

(5 marks)

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SECTION B (35 marks)

15. Four digits are randomly drawn from the seven digits 1 , 2 , 3 , 5 , 6, 7 and 9 . Find the probability that the four digits can form a four-digit even number. (3 marks)

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16. Let α and β be real numbers such that $\begin{cases} 2^{\beta-\alpha} = 16 \\ \log_2(2\alpha+\beta) = 4 \end{cases}$.

(a) Find α and β. (2 marks)

(b) The 1st term and the 2nd term of a geometric sequence are $\log\alpha$ and $\log\beta$ respectively. Find the least value of n such that the sum of the first n terms of the sequence is greater than 1000. (4 marks)

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17. (a) In ΔABC, $BC = a$, $AC = b$ and $AB = c$. Denote the radius of the inscribed circle of ΔABC by r and $s = \frac{a+b+c}{2}$.

(i) Prove that the area of $\Delta ABC = rs$.

(ii) Hence and using Heron's formula,

prove that $r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$.

(4 marks)

(b) The coordinates of the points P and Q are $(12, 0)$ and $(0, 5)$ respectively. Find the equation of the inscribed circle of ΔOPQ, where O is the origin. (3 marks)

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18. Figure 3(a) shows a quadrilateral paper card $ABCD$ with $AB = BC = 16$ cm , $BD = 20$ cm , $\angle DAB = 56°$ and $AD = DC$. The paper card in Figure 3(a) is folded along BD such that the distance between A and C is 15 cm (see Figure 3(b)) .

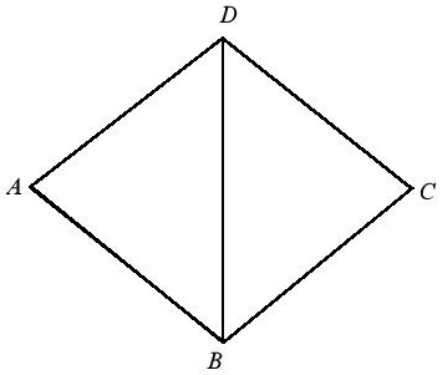

Figure 3 (a)

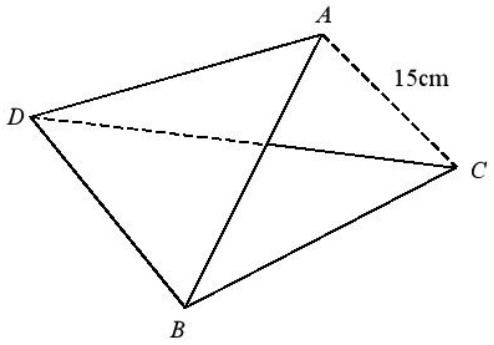

Figure 3 (b)

(a) Find

(i) $\angle ABD$,

(ii) the angle between the plane ABD and the plane BCD.

(5 marks)

(b) Let θ be the angle between AB and the plane BCD. A student claims that $\theta > \angle ABC$.

Is the claim correct? Explain your answer. (3 marks)

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19. Let $f(x) = 3x^2 - 6(k-1)x + 4k^2 - 6k - 6$, where k is a positive constant. P is the vertex of the graph of $y = f(x)$.

(a) Using the method of completing the square, express the coordinates of P in terms of k.

(3 marks)

(b) The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the x-axis and then translating the resulting graph upwards by 6 units. Let Q be the vertex of the graph of $y = g(x)$. Denote the origin by O.

(i) Express the coordinates of Q in terms of k.

(ii) If P and Q lie in the first quadrant, find the range of values of k.

(iii) The coordinates of the point R are $(-1, 3)$. If the y-intercept of the graph of $y = f(x)$ is 34, are P, Q, O and R concyclic? Explain your answer.

(8 marks)

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