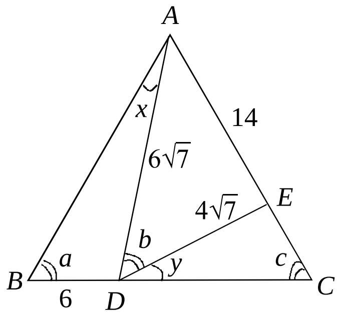
試卷一

	解	分	
1.	$\dfrac{2(m+2n)}{3}=3m-2$		
	$2(m+2n)=3(3m-2)$	1M	
	$2m+4n=9m-6$ $9m-2m=4n+6$	1M	給將 m 放在一邊
	$7m=4n+6$		
	$m=\dfrac{4n+6}{7}$	1A	或等價
	$\dfrac{2}{3}m+\dfrac{4}{3}n=3m-2$	1M	
	$3m-\dfrac{2}{3}m=\dfrac{4}{3}n+2$	1M	給將 m 放在一邊
	$\dfrac{7}{3}m=\dfrac{4}{3}n+2$		
	$m=\dfrac{4}{7}n+\dfrac{6}{7}$	1A	或等價
		---------(3)	
2.	$\dfrac{3}{3x-5}+\dfrac{2}{3-2x}$ | $\dfrac{3}{3x-5}+\dfrac{2}{3-2x}$		
	$=\dfrac{3}{3x-5}-\dfrac{2}{2x-3}$		
	$=\dfrac{3(2x-3)-2(3x-5)}{(3x-5)(2x-3)}$ | $=\dfrac{3(3-2x)+2(3x-5)}{(3x-5)(3-2x)}$	1M	
	$=\dfrac{6x-9-6x+10}{(3x-5)(2x-3)}$ | $=\dfrac{9-6x+6x-10}{(3x-5)(3-2x)}$	1M	
	$=\dfrac{1}{(3x-5)(2x-3)}$ | $=\dfrac{-1}{(3x-5)(3-2x)}$	1A	或等價
		---------(3)	
3.	$\dfrac{(a^{-3})^2}{(a^8b^{-2})^{-3}}$		
	$=\dfrac{a^{-6}}{a^{-24}b^6}$	1M	給 $(xy)^m=x^my^m$ 或 $(x^m)^n=x^{mn}$
	$=\dfrac{a^{24-6}}{b^6}$	1M	給 $z^{-p}=\dfrac{1}{z^p}$ 或 $\dfrac{z^p}{z^q}=z^{p-q}$
	$=\dfrac{a^{18}}{b^6}$	1A	
		---------(3)	

	解	分	備註
4.	(a) $3a^3+a^2b-2ab^2$		
	$=a(3a^2+ab-2b^2)$	1M	
	$=a(3a-2b)(a+b)$	1A	或等價
	(b) $3a^3+a^2b-2ab^2-6a+4b$		
	$=a(3a-2b)(a+b)-6a+4b$	1M	給利用 (a) 的結果
	$=a(3a-2b)(a+b)-2(3a-2b)$		
	$=(3a-2b)(a^2+ab-2)$	1A	或等價
		---------(4)	
5.	(a) 售價 $=\$168\times75\%$	1M	
	$=\$126$	1A	
	(b) 盈利的百分率 $=\dfrac{126-90}{90}\times100\%$	1M	
	$=40\%$	1A	
		---------(4)	
6.	(a) $4(x+2)>\dfrac{11x-5}{5}$		
	$20(x+2)>11x-5$ | $4x+8>\dfrac{11}{5}x-1$		
	$20x+40>11x-5$ | $4x-\dfrac{11}{5}x>-9$		
	$9x>-45$ | $\dfrac{9}{5}x>-9$	1M	給將 x 放在一邊
	$x>-5$ | $x>-5$	1A	
	(b) $5+2x\geq-6$		
	$2x\geq-11$		
	$x\geq-\dfrac{11}{2}$	1A	
	不等式組的解為 $x>-5$ 。		
	最小整數解為 -4 。	1A	
		---------(4)	

		解	分	備註
7.	(a)	A' 的極坐標為 $(24\,,327°)$ 。	1A	接受 $A'(24\,,327°)$ 或 $A'=(24\,,327°)$
	(b)	L 為線段 AA' 的垂直平分線。	1A	
	(c)	設 B 的極坐標為 $(r\,,\theta)$ 。		
		則 $r=2\times 24\cos 30°$	1M	
		$=24\sqrt{3}$ ，		
		$\theta=327°+30°=357°$	1A	
		$\therefore$ B 的極坐標為 $(24\sqrt{3}\,,357°)$ 。		
		$\theta=360°-(30°-27°)=357°$	1A	
			---------(4)	
8.	(a)	$\because\ 3\overset{\frown}{CD}=4\overset{\frown}{AB}$		
		$\therefore \overset{\frown}{CD}:\overset{\frown}{AB}=4:3$		$\overset{\frown}{CD}=\frac{4}{3}\overset{\frown}{AB}$
		設 $\angle CAD=4k$ ， $\angle ADB=3k$ ，	1M	
		則 $4k+3k+96°=180°$	1A	
		$7k=84°$		
		$k=12°$		
		$\angle ADB=3k=36°$	1A	
		設 $\angle ADB=x°$ ，則 $\angle CAD=\frac{4}{3}x°$ ，	1M	設 $\angle ADB=x$
		$x°+\frac{4}{3}x°+96°=180°$	1A	$x+\frac{4}{3}x+96°=180°$
		$\frac{7}{3}x°=84°$		
		$x°=36°$	1A	$x=36°$
		$\therefore\ \angle ADB=36°$		
	(b)	連 CD 。		
		$\angle ADC=90°$		
		$\angle ACD=90°-4\times 12°$	1M	
		$=42°$		
		$\angle ABD=\angle ACD$		
		$=42°$	1A	
		$\angle BDC=90°-36°$	1M	
		$=54°$		
		$\angle BAC=\angle BDC$		
		$=54°$		
		$\angle ABD=96°-54°$		
		$=42°$	1A	
			---------(5)	

解	分	備註
9. (a) 5	1A	
(b) 未知的圓心角 $=360°-108°-72°-126°$ $=54°$		
$\dfrac{9\times108+4\times126+7\times72+54x}{360}=6.25$	1M	
$54x=270$ $x=5$	1A	
(c) 所求概率 $=\dfrac{126+54}{360}$	1M	
$=\dfrac{1}{2}$	1A	
	---------(5)	
10. (a) 設 $\mathrm{h}(x)=ax+bx^2$，其中 a 及 b 為非零常數。	1A	
由 $\mathrm{h}(-2)=8$ 及 $\mathrm{h}(3)=33$，得 $-2a+4b=8$ $a-2b=-4$ ……(1) 又 $3a+9b=33$ $a+3b=11$ ……(2)	1M	給任何一項代換
解 (1)、(2) 兩式，得 $a=2$ ，$b=3$ 。 $\therefore\ \mathrm{h}(x)=2x+3x^2$	1A	給兩項正確
	---------(3)	
(b) $\mathrm{h}(2x)=16$		
$2(2x)+3(2x)^2=16$	1M	
$4x+12x^2=16$ $3x^2+x-4=0$	1A	
$(3x+4)(x-1)=0$ $x=-\dfrac{4}{3}$ 或 $x=1$	1A	給兩項正確
$2(2x)+3(2x)^2=16$ 令 $y=2x$，	1M	
則 $3y^2+2y-16=0$	1A	
$(3y+8)(y-2)=0$ $y=-\dfrac{8}{3}$ 或 $y=2$ $2x=-\dfrac{8}{3}$ 或 $2x=2$		
$x=-\dfrac{4}{3}$ 或 $x=1$	1A	給兩項正確
	---------(3)	

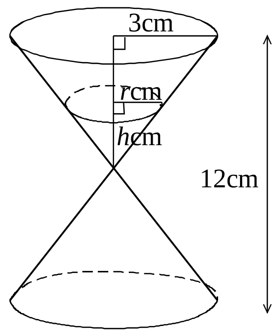
解	分	備註
11.		
		
(a) $\because$ $a=b=c$ (已知)		
$y+b=a+x$ (Δ 外角)		
$\therefore$ $y=x$		
在 ΔABD 及 ΔDCE 中		
$\because$ $a=c$ (已知)		
$x=y$ (已證)		
$\therefore$ $\Delta ABD \sim \Delta DCE$ (等角)		A.A.A 或 A.A.

評分標準：	
情況 1 附有正確理由的任何正確證明。	3
情況 2 未附有正確理由的任何正確證明。	2
情況 3 附有一正確理由和一正確步驟之未完整的證明。	1

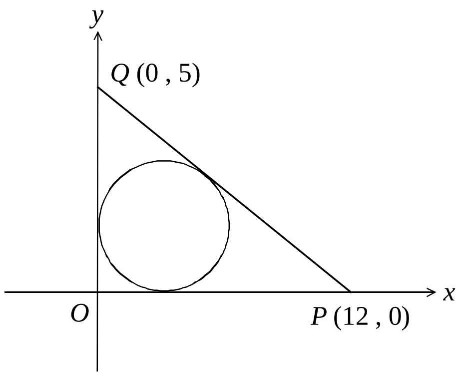
解	分	備註
	---------(3)	
(b) 由 (a) 知 $\dfrac{AD}{DE}=\dfrac{BD}{CE}$ $(=\dfrac{AB}{DC})$		
$\dfrac{6\sqrt{7}}{4\sqrt{7}}=\dfrac{6}{CE}$	1M	
$CE=4$		
$AB=AC=14+4=18$ $(\angle ABD=\angle DCE)$		
又 $\dfrac{18}{DC}=\dfrac{6}{4}$		
$DC=12$		
$\therefore$ $BC=6+12=18$	1A	
$\therefore$ $AB=AC=BC=18$		
因此 ABC 為一等邊三角形。	1A	必須顯示理由
	---------(3)	

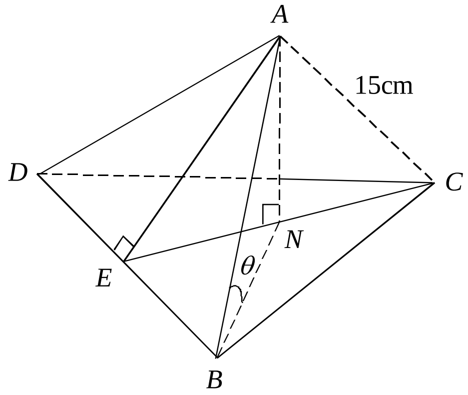
解	分	
12. (a) $a=15$	1A	
$b=\dfrac{28+29}{2}$		
$=28.5$	1A	
$\dfrac{19+20+k}{2}=21$		
$k=3$	1A	
	---------(3)	
(b) 平均值 $=22.5$		
設其餘兩個數據為 p 及 q，		
則 $\dfrac{p+q+23+24}{4}=22.5$	1M	
$p+q=43$	1A	
若果中位數為 21 及有兩個數據為 23 與 24，		
則有 $p<21$ 及 $q<21$	1M	
$p+q<42$		
此與 $p+q=43$ 相矛盾。		
所以前、後兩組的中位數不可能相同。	1A	必須顯示理由
	---------(4)	

		解	分	備註
13.	(a)	設 $p(x)=(x^2-x-6)(ax+b)$，其中 a、b 為常數	1M	給商式 $ax+b$
		則 $p(-1)=-4(-a+b)=4$	1M	給任何一項代換
		$a-b=1$ ……(1)		
		則 $p(2)=-4(2a+b)=-20$		
		$2a+b=5$ ……(2)		
		解 (1)、(2) 兩式，得 $a=2$ ，$b=1$ 。	1A	給兩項正確
		$\therefore$ $p(x)=(x^2-x-6)(2x+1)$		或 $p(x)=2x^3-x^2-13x-6$
		設 $p(x)=ax^3+bx^2+cx+d$ ，其中 a、b、c 及 d 為常數		
		由於 $p(x)$ 可被 $x^2-x-6=(x-3)(x+2)$ 整除，		
		故 $p(3)=27a+9b+3c+d=0$ ……(1)	1M	給任何一項代換
		$p(-2)=-8a+4b-2c+d=0$ ……(2)		
		又 $p(-1)=-a+b-c+d=4$ ……(3)	1M	給任何一項代換
		$p(2)=8a+4b+2c+d=-20$ ……(4)		
		解 (1)、(2)、(3) 及 (4) 得		
		$a=2$ ，$b=-1$ ，$c=-13$ ，$d=-6$	1A	給四項正確
			---------(3)	
	(b)	$p(x)=q(x)$		
		$(x^2-x-6)(2x+1)=(x-3)(2x+k)$	1M	
		$(x-3)(x+2)(2x+1)=(x-3)(2x+k)$		
		$(x-3)(2x^2+3x+2-k)=0$	1A	
		$x=3$ 或 $2x^2+3x+2-k=0$		
		由於方程有三個相異的實數根，		
		故 $\Delta=9-8(2-k)>0$	1M	
		$k>\frac{7}{8}$		
		若果 3 為 $2x^2+3x+2-k=0$ 的根，		
		則 $29-k=0$	1M	給將 3 代入方程
		$k=29$		
		$\therefore$ $k>\frac{7}{8}$ 且 $k\neq 29$	1A	
			---------(5)	

解	分	
14. (a) 由相似三角形，得		
$\dfrac{r}{3}=\dfrac{h}{6}$ 	1M	
$r=\dfrac{h}{2}$		
沙的體積		
$=\dfrac{1}{3}\pi r^2 h$		
$=\dfrac{1}{3}\pi(\dfrac{h}{2})^2 h$	1M	1M 給代入
$=\dfrac{\pi}{12}h^3 \quad (\text{cm}^3)$	1A	
	---------(3)	
(b) (i) $\dfrac{\pi}{12}h^3=\dfrac{4\pi}{45}\times 60$	1M	
$h^3=64$		
$h=4$	1A	
(ii) 設此時沙的高度為 k cm，		
則 $\dfrac{\frac{1}{3}\pi\cdot 3^2\cdot 6-\frac{\pi}{12}\cdot 4^3}{\frac{1}{3}\pi\cdot 3^2\cdot 6}=(\dfrac{6-k}{6})^3$	1M+1A	
$\dfrac{6-k}{6}=\dfrac{\sqrt[3]{19}}{3}$		
$k=6-2(\sqrt[3]{19})$		
≈ 0.663196702		接受答案準確至 0.663
<1		
$\therefore$ 同意該宣稱。	1A	必須顯示理由
	---------(5)	

解	分	
15. 所求概率 $=\dfrac{C_1^2C_3^5+C_2^2C_2^5}{C_4^7}$	1M+1M	1M 給分子，1M 給分母
$=\dfrac{6}{7}$	1A	接受答案準確至 0.857
所求概率 $=1-\dfrac{C_4^5}{C_4^7}$	1M+1M	1M 給 $1-P$ ，1M 給 $\dfrac{C_4^5}{C_4^7}$
$=1-\dfrac{1}{7}$		
$=\dfrac{6}{7}$	1A	
	---------(3)	
16. (a) $\begin{cases}2^{\beta-\alpha}=16\\ \log_2(2\alpha+\beta)=4\end{cases}$		
$\begin{cases}\beta-\alpha=4\\ 2\alpha+\beta=16\end{cases}$	1A	給任何一項
解之得 $\alpha=4$ ， $\beta=8$	1A	給兩項正確
	---------(2)	
(b) 第一項 $T_1=\log 4=2\log 2$		
第二項 $T_2=\log 8=3\log 2$		
公比 $=\dfrac{3\log 2}{2\log 2}=\dfrac{3}{2}$	1M	
$\dfrac{2\log 2[(\frac{3}{2})^n-1]}{\frac{3}{2}-1}>1000$	1M	
$(\frac{3}{2})^n-1>\dfrac{250}{\log 2}$		$(\frac{3}{2})^n-1>830.4820237$
$(\frac{3}{2})^n>\dfrac{250}{\log 2}+1$		$(\frac{3}{2})^n>831.4820237$
$n\log\dfrac{3}{2}>\log(\dfrac{250}{\log 2}+1)$	1M	$n\log 1.5>2.919852864$
$n>\dfrac{\log(\frac{250}{\log 2}+1)}{\log\frac{3}{2}}$		
$n>16.58147531$		$n>16.58147531$
n 的最小值為 17。	1A	
	---------(4)	

解	分	備註
17. (a) (i) ΔABC 的面積		
$= \frac{ar}{2} + \frac{br}{2} + \frac{cr}{2}$	1M	
$= \frac{r}{2}(a+b+c)$		
$= rs$	1	必須顯示理由
(ii) 又 ΔABC 的面積		
$= \sqrt{s(s-a)(s-b)(s-c)}$		
$\therefore\ rs = \sqrt{s(s-a)(s-b)(s-c)}$	1M	
$r = \frac{1}{s}\sqrt{s(s-a)(s-b)(s-c)}$		
$= \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$	1	必須顯示理由
	---------(4)	
(b) 		
$OP = 12$ ， $OQ = 5$		
$PQ = \sqrt{12^2 + 5^2} = 13$		
半周界 $s = \frac{12+5+13}{2}$	1M	
$= 15$		
$\therefore\ r = \sqrt{\frac{(15-12)(15-5)(15-13)}{15}}$	1M	
$= 2$		
圓心 $= (2, 2)$		
內切圓的方程為 $(x-2)^2 + (y-2)^2 = 4$ 。	1A	接受 $(x-2)^2 + (y-2)^2 = 2^2$ 或 $x^2 + y^2 - 4x - 4y + 4 = 0$
	---------(3)	

	解	分	
18.	(a) (i) $\frac{20}{\sin 56°}=\frac{16}{\sin\angle ADB}$	1M	
	$\angle ADB\approx 41.54668313°$		
	$\angle ABD\approx 82.45331687°$		
	$\approx 82.5°$	1A	接受答案準確至 82.5°
	(ii) 		
	設 E 為 A 至 BD 的垂足。		
	由於 $\Delta DBA\cong\Delta DBC$		
	故 $CE\perp BD$ 。		
	$\therefore$ $\angle AEC$ 為所求的角。	1M	
	$AE\approx 16\sin 82.45331687°$		
	$=15.86141093$		
	$\cos\angle AEC\approx\frac{15.86141093^2+15.86141093^2-15^2}{2(15.86141093)^2}$	1M	
	$\angle AEC\approx 56.43835278°$		
	所求交角為 56.4° 。	1A	接受答案準確至 56.4°
	留意 $EA=EC$		
	$\sin(\frac{\angle AEC}{2})=\frac{\frac{1}{2}AC}{AE}$	1M	
	$\approx\frac{7.5}{15.86141093}$		
	$\angle AEC\approx 56.4°$	1A	接受答案準確至 56.4°
		--------(5)	

解	分	
(b) 設 N 為由 A 至平面 BCD 的垂足。		
留意 N 在 CE 上。		
$AN = AE\sin\angle AEC$		
$\approx 15.86141093\sin 56.43835278°$		
≈ 13.21717866		
$\sin\theta = \dfrac{AN}{AB}$		
$= \dfrac{13.21717866}{16}$		
$\theta = 55.69749604°$	1A	接受答案準確至 55.7°
又 $\sin\dfrac{\angle ABC}{2} = \dfrac{\frac{1}{2}AC}{AB}$	1M	
$= \dfrac{7.5}{16}$		
$\angle ABC \approx 55.90637377°$		接受答案準確至 55.9°
$> \theta$		
因此，不同意該宣稱。	1A	必須顯示理由
$\cos\angle ABC = \dfrac{16^2+16^2-15^2}{2(16)(16)}$	1M	
$\angle ABC \approx 55.90637377°$		接受答案準確至 55.9°
$> \theta$		
因此，不同意該宣稱。	1A	必須顯示理由
	--------(3)	

解	分	
19. (a) $f(x)=3x^2-6(k-1)x+4k^2-6k-6$		
$=3[x^2-2(k-1)x+(k-1)^2]-3(k-1)^2+4k^2-6k-6$	1M	給配方法
$=3(x-(k-1))^2+k^2-9$	1M	
P 的坐標為 $(k-1\,,k^2-9)$ 。	1A	接受 $P(k-1\,,k^2-9)$ 或 $P=(k-1\,,k^2-9)$
	---------(3)	
(b) (i) Q 的坐標為 $(k-1\,,15-k^2)$	1M	
(ii) $\because$ P 及 Q 位於第一象限內		
$\therefore$ $k-1>0$ 及 $k^2-9>0$ 及 $15-k^2>0$	1M	
$k>1$ 及 ($k<-3$ 或 $k>3$) 及 $-\sqrt{15}<k<\sqrt{15}$	1A	
答： $3<k<\sqrt{15}$	1A	
(iii) $4k^2-6k-6=34$		
$2k^2-3k-20=0$		
$(k-4)(2k+5)=0$		
$\therefore$ $k=4$ 或 $k=-\dfrac{5}{2}$ (捨)	1A	
P 點坐標為 $(3\,,7)$		
Q 點坐標為 $(3\,,-1)$		
R 點坐標為 $(-1\,,3)$		
$\because$ $m_{PR}\cdot m_{QR}=(\dfrac{7-3}{3+1})(\dfrac{3+1}{-1-3})=-1$	1M	
$\therefore$ $\angle PRQ=90^\circ$		
又 $m_{PO}\cdot m_{QO}=(\dfrac{7-0}{3-0})(\dfrac{-1-0}{3-0})\neq -1$	1M	
$\therefore$ $\angle POQ\neq 90^\circ$		
$\therefore$ $\angle PRQ\neq\angle POQ$		
因此， P 、 Q 、 O 與 R 四點不共圓。	1A	必須顯示理由
設通 P 、 Q 及 O 的圓方程為		
$x^2+y^2+Dx+Ey+F=0$		
代入 $(0\,,0)$ 得 $F=0$		
代入 $(3\,,7)$ 及 $(3\,,-1)$ 得		
$9+49+3D+7E=0$ $9+1+3D-E=0$	1M	給任何一項
解之，得 $D=-\dfrac{16}{3}$ ， $E=-6$		
圓方程為 $x^2+y^2-\dfrac{16}{3}x-6y=0$		
即 $3x^2+3y^2-16x-18y=0$		
代入 $R(-1\,,3)$		
$\because$ $3+27+16-54\neq 0$	1M	
$\therefore$ 通過 P 、 Q 及 O 的圓不通過 R 點。		
因此， P 、 Q 、 O 與 R 四點不共圓。	1A	必須顯示理由
	--------(8)	

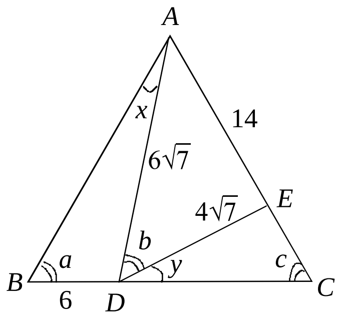
Paper I

	Solution	Marks	Remarks
1.	$\dfrac{2(m+2n)}{3}=3m-2$		
	$2(m+2n)=3(3m-2)$	1M	
	$2m+4n=9m-6$		
	$9m-2m=4n+6$	1M	for putting m on one side
	$7m=4n+6$		
	$m=\dfrac{4n+6}{7}$	1A	or equivalent
	$\dfrac{2}{3}m+\dfrac{4}{3}n=3m-2$	1M	
	$3m-\dfrac{2}{3}m=\dfrac{4}{3}n+2$	1M	for putting m on one side
	$\dfrac{7}{3}m=\dfrac{4}{3}n+2$		
	$m=\dfrac{4}{7}n+\dfrac{6}{7}$	1A	or equivalent
		---------(3)	
2.	$\dfrac{3}{3x-5}+\dfrac{2}{3-2x}$ $=\dfrac{3}{3x-5}-\dfrac{2}{2x-3}$		
	$=\dfrac{3(2x-3)-2(3x-5)}{(3x-5)(2x-3)}$	1M	
	$=\dfrac{6x-9-6x+10}{(3x-5)(2x-3)}$	1M	
	$=\dfrac{1}{(3x-5)(2x-3)}$	1A	or equivalent
	$\dfrac{3}{3x-5}+\dfrac{2}{3-2x}$		
	$=\dfrac{3(3-2x)+2(3x-5)}{(3x-5)(3-2x)}$	1M	
	$=\dfrac{9-6x+6x-10}{(3x-5)(3-2x)}$	1M	
	$=\dfrac{-1}{(3x-5)(3-2x)}$	1A	or equivalent
		---------(3)	
3.	$\dfrac{(a^{-3})^2}{(a^8b^{-2})^{-3}}$		
	$=\dfrac{a^{-6}}{a^{-24}b^6}$	1M	for $(xy)^m=x^my^m$ or $(x^m)^n=x^{mn}$
	$=\dfrac{a^{24-6}}{b^6}$	1M	for $z^{-p}=\dfrac{1}{z^p}$ or $\dfrac{z^p}{z^q}=z^{p-q}$
	$=\dfrac{a^{18}}{b^6}$	1A	
		---------(3)	

	Solution	Marks	Remarks
4.	(a) $3a^3+a^2b-2ab^2$		
	$=a(3a^2+ab-2b^2)$	1M	
	$=a(3a-2b)(a+b)$	1A	or equivalent
	(b) $3a^3+a^2b-2ab^2-6a+4b$		
	$=a(3a-2b)(a+b)-6a+4b$	1M	for using the result of (a)
	$=a(3a-2b)(a+b)-2(3a-2b)$		
	$=(3a-2b)(a^2+ab-2)$	1A	or equivalent
		---------(4)	
5.	(a) The selling price $=\$168\times 75\%$	1M	
	$=\$126$	1A	
	(b) The percentage profit $=\dfrac{126-90}{90}\times 100\%$	1M	
	$=40\%$	1A	
		---------(4)	
6.	(a) $4(x+2)>\dfrac{11x-5}{5}$		
	$20(x+2)>11x-5$ | $4x+8>\dfrac{11}{5}x-1$		
	$20x+40>11x-5$ | $4x-\dfrac{11}{5}x>-9$		
	$9x>-45$ | $\dfrac{9}{5}x>-9$	1M	for putting x on one side
	$x>-5$ | $x>-5$	1A	
	(b) $5+2x\geq -6$		
	$2x\geq -11$		
	$x\geq -\dfrac{11}{2}$	1A	
	Thus, the required range is $x>-5$.		
	The least integer is -4.	1A	
		---------(4)	

		Solution	Marks	Remarks
7.	(a)	The polar coordinates of A' are $(24, 327°)$.	1A	accept $A'(24, 327°)$ or $A' = (24, 327°)$
	(b)	L is the perpendicular bisector of the line segment AA' .	1A	
	(c)	Let (r, θ) be the polar coordinates of B .		
		Then $r = 2 \times 24 \cos 30°$	1M	
		$= 24\sqrt{3}$,		
		$\theta = 327° + 30° = 357°$	1A	
		$\therefore$ The polar coordinates of B are $(24\sqrt{3}, 357°)$.		
		$\theta = 360° - (30° - 27°) = 357°$	1A	
			---------(4)	
8.	(a)	$\because 3\overset{\frown}{CD} = 4\overset{\frown}{AB}$		
		$\therefore \overset{\frown}{CD} : \overset{\frown}{AB} = 4 : 3$		$\overset{\frown}{CD} = \frac{4}{3}\overset{\frown}{AB}$
		Let $\angle CAD = 4k$, $\angle ADB = 3k$,	1M	
		then $4k + 3k + 96° = 180°$	1A	
		$7k = 84°$		
		$k = 12°$		
		$\angle ADB = 3k = 36°$	1A	
		Let $\angle ADB = x°$, then $\angle CAD = \frac{4}{3}x°$,	1M	Let $\angle ADB = x$
		$x° + \frac{4}{3}x° + 96° = 180°$	1A	$x + \frac{4}{3}x + 96° = 180°$
		$\frac{7}{3}x° = 84°$		
		$x° = 36°$	1A	$x = 36°$
		$\therefore \angle ADB = 36°$		
	(b)	Join CD .		
		$\angle ADC = 90°$		
		$\angle ACD = 90° - 4 \times 12°$	1M	
		$= 42°$		
		$\angle ABD = \angle ACD$		
		$= 42°$	1A	
		$\angle BDC = 90° - 36°$	1M	
		$= 54°$		
		$\angle BAC = \angle BDC$		
		$= 54°$		
		$\angle ABD = 96° - 54°$		
		$= 42°$	1A	
			---------(5)	

Solution	Marks	Remarks
9. (a) 5	1A	
(b) Unknown angle at the centre		
$=360^\circ-108^\circ-72^\circ-126^\circ$		
$=54^\circ$		
$\frac{9\times108+4\times126+7\times72+54x}{360}=6.25$	1M	
$54x=270$		
$x=5$	1A	
(c) The required probability		
$=\frac{126+54}{360}$	1M	
$=\frac{1}{2}$	1A	
	---------(5)	
10. (a) Let $\mathrm{h}(x)=ax+bx^2$, where a and b are non-zero constants.	1A	
From $\mathrm{h}(-2)=8$ and $\mathrm{h}(3)=33$, we have		
$-2a+4b=8$	1M	for either substitution
$a-2b=-4$ ……(1)		
and $3a+9b=33$		
$a+3b=11$ ……(2)		
By solving (1) and (2), we have $a=2$, $b=3$.	1A	for both correct
$\therefore\ \mathrm{h}(x)=2x+3x^2$		
	---------(3)	
(b) $\mathrm{h}(2x)=16$		
$2(2x)+3(2x)^2=16$	1M	
$4x+12x^2=16$		
$3x^2+x-4=0$	1A	
$(3x+4)(x-1)=0$		
$x=-\frac{4}{3}$ or $x=1$	1A	for both correct
$2(2x)+3(2x)^2=16$	1M	
Let $y=2x$,		
then $3y^2+2y-16=0$	1A	
$(3y+8)(y-2)=0$		
$y=-\frac{8}{3}$ or $y=2$		
$2x=-\frac{8}{3}$ or $2x=2$		
$x=-\frac{4}{3}$ or $x=1$	1A	for both correct
	---------(3)	

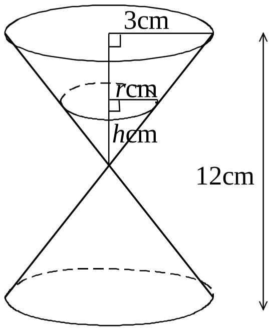
Solution	Marks	Remarks
11.		
		
(a) $\because\ a=b=c$ (given)		
$y+b=a+x$ (ext. $\angle$ of Δ)		
$\therefore\ y=x$		
In ΔABC and ΔDCE		
$\because\ a=c$ (given)		
$x=y$ (proved)		
$\therefore\ \Delta ABD \sim \Delta DCE$ (equiangular)		A.A.A or A.A.

Marking Scheme：	
Case 1 Any correct proof with correct reasons.	3
Case 2 Any correct proof without reasons.	2
Case 3 Any relevant argument with correct reason.	1

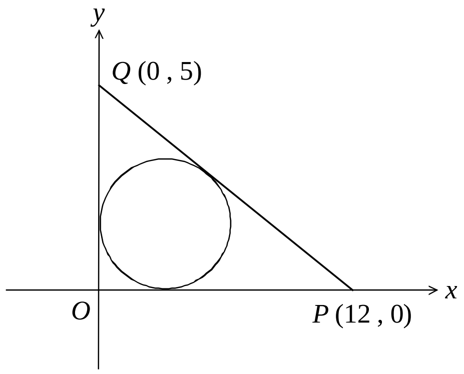
Solution	Marks	Remarks
	---------(3)	
(b) From (a), we have $\dfrac{AD}{DE}=\dfrac{BD}{CE}$ $\left(=\dfrac{AB}{DC}\right)$		
$\dfrac{6\sqrt{7}}{4\sqrt{7}}=\dfrac{6}{CE}$	1M	
$CE=4$		
$AB=AC=14+4=18$ $(\angle ABD=\angle DCE)$		
and $\dfrac{18}{DC}=\dfrac{6}{4}$		
$DC=12$		
$\therefore\ BC=6+12=18$	1A	
$\therefore\ AB=AC=BC=18$		
Thus, ABC is an equilateral triangle.	1A	f.t.
	---------(3)	

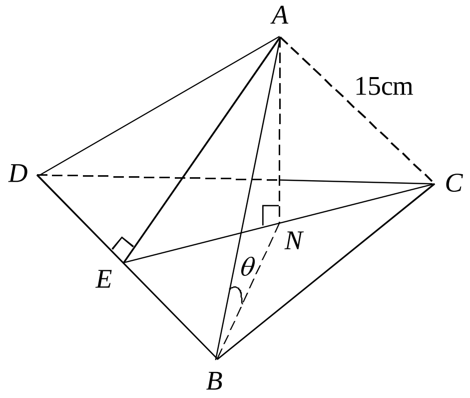
		Solution	Marks	
12.	(a)	$a = 15$	1A	
		$b = \dfrac{28+29}{2}$		
		$= 28.5$	1A	
		$\dfrac{19+20+k}{2} = 21$		
		$k = 3$	1A	
			---------(3)	
	(b)	The mean $= 22.5$		
		Let p and q be the remaining two data,		
		then $\dfrac{p+q+23+24}{4} = 22.5$	1M	
		$p+q = 43$	1A	
		If the median is 21 and there are two data which are 23 and 24		
		So, we have $p < 21$ and $q < 21$	1M	
		$p+q < 42$		
		It is impossible since $p+q = 43$.		
		Thus, the medians of the former and the latter data are not the same.	1A	f.t.
			---------(4)	

Solution	Marks	Remarks
13. (a) Let $p(x)=(x^2-x-6)(ax+b)$, where a and b are constants.	1M	for quotient $ax+b$
Then $p(-1)=-4(-a+b)=4$	1M	for either substitution
$a-b=1$ ……(1)		
Then $p(2)=-4(2a+b)=-20$		
$2a+b=5$ ……(2)		
By solving (1) and (2), we have $a=2$, $b=1$.	1A	for both correct
$\therefore$ $p(x)=(x^2-x-6)(2x+1)$		or $p(x)=2x^3-x^2-13x-6$
Let $p(x)=ax^3+bx^2+cx+d$, where a, b, c and d are constants		
Since $p(x)$ is divisible by $x^2-x-6=(x-3)(x+2)$,		
so we have $p(3)=27a+9b+3c+d=0$ ……(1)	1M	for either substitution
$p(-2)=-8a+4b-2c+d=0$ ……(2)		
and $p(-1)=-a+b-c+d=4$ ……(3)	1M	for either substitution
$p(2)=8a+4b+2c+d=-20$ ……(4)		
By solving (1), (2), (3) and (4), we have		
$a=2$, $b=-1$, $c=-13$, $d=-6$	1A	for four correct
	---------(3)	
(b) $p(x)=q(x)$		
$(x^2-x-6)(2x+1)=(x-3)(2x+k)$	1M	
$(x-3)(x+2)(2x+1)=(x-3)(2x+k)$		
$(x-3)(2x^2+3x+2-k)=0$	1A	
$x=3$ or $2x^2+3x+2-k=0$		
Since the equation has three distinct real roots,		
So, we have $\Delta=9-8(2-k)>0$	1M	
$k>\frac{7}{8}$		
If 3 is the root of $2x^2+3x+2-k=0$,		
then $29-k=0$	1M	for substituting 3 into the equation
$k=29$		
$\therefore$ $k>\frac{7}{8}$ and $k\neq 29$	1A	
	---------(5)	

Solution	Marks	
14. (a) Consider the similar triangles, we have		
$\dfrac{r}{3}=\dfrac{h}{6}$	1M	
		
$r=\dfrac{h}{2}$		
The volume of the sand		
$=\dfrac{1}{3}\pi r^2 h$		
$=\dfrac{1}{3}\pi(\dfrac{h}{2})^2 h$	1M	1M for substitution
$=\dfrac{\pi}{12}h^3 \quad (\text{cm}^3)$	1A	
	---------(3)	
(b) (i) $\dfrac{\pi}{12}h^3=\dfrac{4\pi}{45}\times 60$	1M	
$h^3=64$		
$h=4$	1A	
(ii) Let k cm be the height of the sand,		
we have $\dfrac{\frac{1}{3}\pi\cdot 3^2\cdot 6-\frac{\pi}{12}\cdot 4^3}{\frac{1}{3}\pi\cdot 3^2\cdot 6}=(\dfrac{6-k}{6})^3$	1M+1A	
$\dfrac{6-k}{6}=\dfrac{\sqrt[3]{19}}{3}$		
$k=6-2(\sqrt[3]{19})$		
≈ 0.663196702		accept answer correct to 0.663
<1		
$\therefore$ The claim is agreed.	1A	f.t.
	---------(5)	

	Solution	Marks	
15.	The required probability $=\dfrac{C_1^2C_3^5+C_2^2C_2^5}{C_4^7}$	1M+1M	1M for numerator, 1M for denominator
	$=\dfrac{6}{7}$	1A	accept answer correct to 0.857
	The required probability $=1-\dfrac{C_4^5}{C_4^7}$	1M+1M	1M for $1-P$, 1M for $\dfrac{C_4^5}{C_4^7}$
	$=1-\dfrac{1}{7}$		
	$=\dfrac{6}{7}$	1A	
		---------(3)	
16.	(a) $\begin{cases}2^{\beta-\alpha}=16\\ \log_2(2\alpha+\beta)=4\end{cases}$		
	$\begin{cases}\beta-\alpha=4\\ 2\alpha+\beta=16\end{cases}$	1A	either one
	Solving, $\alpha=4$, $\beta=8$	1A	for both correct
		---------(2)	
	(b) The 1st term $T_1=\log 4=2\log 2$		
	The 2nd term $T_2=\log 8=3\log 2$		
	The common ratio $=\dfrac{3\log 2}{2\log 2}=\dfrac{3}{2}$	1M	
	$\dfrac{2\log 2[(\frac{3}{2})^n-1]}{\frac{3}{2}-1}>1000$	1M	
	$(\frac{3}{2})^n-1>\dfrac{250}{\log 2}$		$(\frac{3}{2})^n-1>830.4820237$
	$(\frac{3}{2})^n>\dfrac{250}{\log 2}+1$		$(\frac{3}{2})^n>831.4820237$
	$n\log\dfrac{3}{2}>\log(\dfrac{250}{\log 2}+1)$	1M	$n\log 1.5>2.919852864$
	$n>\dfrac{\log(\frac{250}{\log 2}+1)}{\log\frac{3}{2}}$		
	$n>16.58147531$		$n>16.58147531$
	The least value of n is 17 .	1A	
		---------(4)	

Solution	Marks	Remarks
17. (a) (i) The area of ΔABC		
$= \frac{ar}{2} + \frac{br}{2} + \frac{cr}{2}$	1M	
$= \frac{r}{2}(a+b+c)$		
$= rs$	1	f.t.
(ii) The area of ΔABC		
$= \sqrt{s(s-a)(s-b)(s-c)}$		
$\therefore\ rs = \sqrt{s(s-a)(s-b)(s-c)}$	1M	
$r = \frac{1}{s}\sqrt{s(s-a)(s-b)(s-c)}$		
$= \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$	1	f.t.
	---------(4)	
(b) 		
$OP = 12$ ， $OQ = 5$		
$PQ = \sqrt{12^2 + 5^2} = 13$		
The semi-perimeter $s = \frac{12+5+13}{2}$	1M	
$= 15$		
$\therefore\ r = \sqrt{\frac{(15-12)(15-5)(15-13)}{15}}$	1M	
$= 2$		
The centre $= (2 , 2)$		
The equation of the inscribed circle is $(x-2)^2 + (y-2)^2 = 4$.	1A	accept $(x-2)^2 + (y-2)^2 = 2^2$ or $x^2 + y^2 - 4x - 4y + 4 = 0$
	---------(3)	

Solution	Marks	
18. (a) (i) $\dfrac{20}{\sin 56^\circ} = \dfrac{16}{\sin \angle ADB}$	1M	
$\angle ADB \approx 41.54668313^\circ$		
$\angle ABD \approx 82.45331687^\circ$		
$\approx 82.5^\circ$	1A	accept answer correct to 82.5°
(ii) 		
Let E be the foot of the perpendicular from A to BD .		
Since $\Delta DBA \cong \Delta DBC$		
$CE \perp BD$.		
$\therefore$ $\angle AEC$ is the required angle.	1M	
$AE \approx 16 \sin 82.45331687^\circ$		
$= 15.86141093$		
$\cos \angle AEC \approx \dfrac{15.86141093^2 + 15.86141093^2 - 15^2}{2(15.86141093)^2}$	1M	
$\angle AEC \approx 56.43835278^\circ$		
The required angle is 56.4° .	1A	accept answer correct to 56.4°
Note that $EA = EC$		
$\sin(\dfrac{\angle AEC}{2}) = \dfrac{\frac{1}{2}AC}{AE}$	1M	
$\approx \dfrac{7.5}{15.86141093}$		
$\angle AEC \approx 56.4^\circ$	1A	accept answer correct to 56.4°
	--------(5)	

Solution	Marks	
(b) Let N be the foot of the perpendicular from A to the plane BCD		
Note that N is lying on CE .		
$AN = AE\sin\angle AEC$		
$\approx 15.86141093\sin 56.43835278^\circ$		
≈ 13.21717866		
$\sin\theta = \dfrac{AN}{AB}$		
$= \dfrac{13.21717866}{16}$		
$\theta = 55.69749604^\circ$	1A	accept answer correct to 55.7°
$\sin\dfrac{\angle ABC}{2} = \dfrac{\frac{1}{2}AC}{AB}$	1M	
$= \dfrac{7.5}{16}$		
$\angle ABC \approx 55.90637377^\circ$		accept answer correct to 55.9°
$> \theta$		
Thus, the claim is disagreed.	1A	f.t.
$\cos\angle ABC = \dfrac{16^2 + 16^2 - 15^2}{2(16)(16)}$	1M	
$\angle ABC \approx 55.90637377^\circ$		accept answer correct to 55.9°
$> \theta$		
Thus, the claim is disagreed.	1A	f.t.
	--------(3)	

Solution	Marks	
19. (a) $f(x) = 3x^2 - 6(k-1)x + 4k^2 - 6k - 6$		
$= 3[x^2 - 2(k-1)x + (k-1)^2] - 3(k-1)^2 + 4k^2 - 6k - 6$	1M	for completing the square.
$= 3(x-(k-1))^2 + k^2 - 9$	1M	
The coordinates of P are $(k-1\,,\,k^2-9)$.	1A	accept $P(k-1\,,\,k^2-9)$ or $P = (k-1\,,\,k^2-9)$
	---------(3)	
(b) (i) The coordinates of Q are $(k-1\,,\,15-k^2)$	1M	
(ii) $\because$ P and Q lie in the first quadrant		
$\therefore$ $k-1>0$ and $k^2-9>0$ and $15-k^2>0$	1M	
$k>1$ and ($k<-3$ or $k>3$) and $-\sqrt{15}<k<\sqrt{15}$	1A	
Answer： $3<k<\sqrt{15}$	1A	
(iii) $4k^2-6k-6=34$		
$2k^2-3k-20=0$		
$(k-4)(2k+5)=0$		
$\therefore$ $k=4$ or $k=-\dfrac{5}{2}$ (rejected)	1A	
The coordinates of P are $(3\,,\,7)$		
The coordinates of Q are $(3\,,\,-1)$		
The coordinates of R are $(-1\,,\,3)$		
$\because$ $m_{PR}\cdot m_{QR} = (\dfrac{7-3}{3+1})(\dfrac{3+1}{-1-3}) = -1$	1M	
$\therefore$ $\angle PRQ = 90°$		
And $m_{PO}\cdot m_{QO} = (\dfrac{7-0}{3-0})(\dfrac{-1-0}{3-0}) \neq -1$	1M	
$\therefore$ $\angle POQ \neq 90°$		
$\therefore$ $\angle PRQ \neq \angle POQ$		
Thus, P , Q , O and R are not concyclic.	1A	f.t.
Let $x^2+y^2+Dx+Ey+F=0$ be the equation of the circle passing through P , Q and O		
Sub. $(0\,,\,0)$, we have $F=0$		
Sub. $(3\,,\,7)$ and $(3\,,\,-1)$, we have		
$9+49+3D+7E=0$ $9+1+3D-E=0$	1M	either one
Solving, we have $D=-\dfrac{16}{3}\,,\,E=-6$		
The equation of the circle is $x^2+y^2-\dfrac{16}{3}x-6y=0$		
i.e. $3x^2+3y^2-16x-18y=0$		
Sub. $R(-1\,,\,3)$		
$\because$ $3+27+16-54\neq 0$	1M	
$\therefore$ The circle which passes through P , Q and O does not pass through the point R .		
Thus, P , Q , O and R are not concyclic.	1A	f.t.
	---------(8)	

2019 / 20 數學 必修部分

試卷二 解答

答案 (打橫看)

D A C D C　　C B D A B　　B A B C C　　D A B D A
C B B A D　　D C C A B　　D A C D A　　B C C B D
A B D D A

1. [D]

$$(\frac{9}{27^{n-1}})^4=(\frac{3^2}{(3^3)^{n-1}})^4=\frac{3^8}{(3^{3n-3})^4}$$
$$=\frac{3^8}{3^{12n-12}}=3^{20-12n}$$

2. [A]

$$\begin{aligned}&m^2+2m-4n^2-4n\\&=m^2-4n^2+2m-4n\\&=(m+2n)(m-2n)+2(m-2n)\\&=(m-2n)(m+2n+2)\end{aligned}$$

3. [C]

$$\begin{cases}3x-2y=11 & \cdots\cdots(1)\\ x+4y-2=11 & \cdots\cdots(2)\end{cases}$$

$(2)\times3$ ，得 $3x+12y=39\quad\cdots\cdots(3)$

$(1)-(2)$ ，得 $-14y=-28$

$$y=2$$

代入 (1) 得 $3x-4=11$ ，　$x=5$ 。

$\therefore\ x-y=5-2=3$

4. [D]

$$(3x+4)(x-\alpha)\equiv3x^2+\beta(x+1)$$
$$3x^2+(4-3\alpha)x-4\alpha\equiv3x^2+\beta x+\beta$$
$$\begin{cases}\beta=4-3\alpha\\ \beta=-4\alpha\end{cases}$$
$$4-3\alpha=-4\alpha$$
$$\alpha=-4\text{ ，}\beta=16$$

5. [C]

$$k = \frac{6}{5-h} - 3$$

$$k + 3 = \frac{6}{5-h}$$

$$5 - h = \frac{6}{k+3}$$

$$h = 5 - \frac{6}{k+3}$$

$$= \frac{5k+9}{k+3}$$

6. [C]

$$\sqrt{2019} + \sqrt{2020} \approx 89.87769399$$
$$= 89.88 \quad \text{(準確至四位有效數字)}$$

7. [B]

$$\frac{3-2x}{4} < -1 \quad \text{或} \quad 3x + 7 \geq 1$$

$$3 - 2x < -4 \quad \text{或} \quad 3x \geq -6$$

$$-2x < -7 \quad \text{或} \quad x \geq -2$$

$$x > \frac{7}{2} \quad \text{或} \quad x \geq -2$$

$\therefore \quad x \geq -2$

答 ： $x = -2$

8. [D]

$$\mathrm{f}(x) = 3x^2 - 4x - 1$$

$$\mathrm{f}(2c-1) = 3(2x-1)^2 - 4(2c-1) - 1$$

$$= 3(4c^2 - 4c + 1) - 8c + 4 - 1$$

$$= 12c^2 - 20c + 6$$

9. [A]

$$9x^2 + (k+1)x + 1 = 0$$

$$\Delta = (k+1)^2 - 36 = 0$$

$$k + 1 = \pm 6$$

$k = 5$ (捨) 或 $k = -7$

10. [B]

頂點 $=(p\,,\,pq)$ 。

由圖知 p 及 pq 均為負值。

$\therefore$ $p<0$ 及 $q>0$ 。

11. [B]

$$50000(1+\frac{x}{2}\%)^8=56325$$

$$(1+\frac{x}{200})^8=\frac{56325}{50000}$$

$$1+\frac{x}{200}=\sqrt[8]{\frac{56325}{50000}}$$

$$x=200(\sqrt[8]{\frac{56325}{50000}}-1)$$

$$=3$$

12. [A]

$$9k^2\text{ cm}^2=22500\text{ m}^2$$

$$9k^2\text{ cm}^2=22500\times100^2\text{ cm}^2$$

$$9k^2=22500\times100^2$$

$$k=5000$$

13. [B]

$z=\dfrac{k\sqrt{x}}{y^2}$ ， k 為常數

$$\frac{zy^2}{\sqrt{x}}=k$$

$$\frac{z^2y^4}{x}=k^2$$

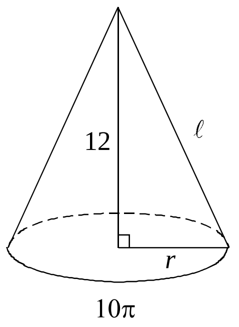
$$\frac{x}{y^4z^2}=\frac{1}{k^2}=\text{常數}$$

14. [C]

點子數目 $=5+8+12+16+20+24+28+32$

$=145$

15. [C]



$2\pi r=10\pi$

$r=5$

$\ell=\sqrt{5^5+12^2}=13$

總表面面積

$=\pi\times5^2+\pi\times5\times13$

$=90\pi$ (cm^2)

16. [D]

$\Delta BCE\sim\Delta FDE$

ΔBCE 面積 ： ΔDEF 面積 $=2^2:3^2=4:9$

設 ΔBCE 面積 $=x\text{ cm}^2$，

則 ΔDEF 面積 $=\dfrac{9}{4}x\text{ cm}^2$。

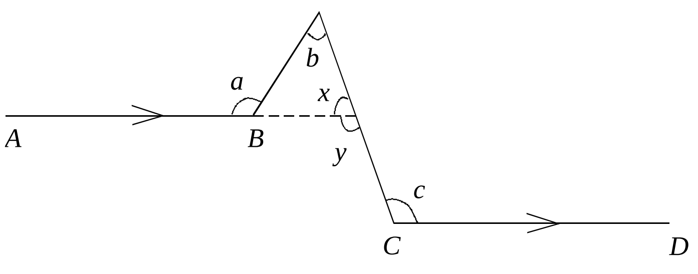
$\therefore\quad x+\dfrac{9}{4}x=78$

$\dfrac{13}{4}x=78$

$x=24$

ΔABE 的面積 $=24\times\dfrac{5}{2}=60$ (cm^2)

17. [A]



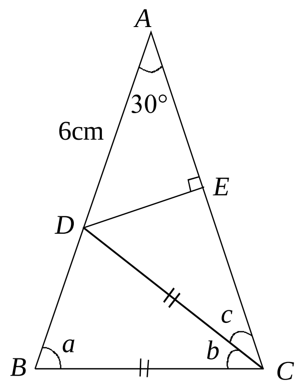
$x=a-b$

$y=180^\circ-x=180^\circ-a+b$

$c=y=180^\circ-a+b$

$\therefore\quad a-b+c=180^\circ$

18. [B]



$a=\dfrac{180°-30°}{2}=75°$

$b=180°-2\times 75°=30°$

$c=75°-30°=45°$

$EC=ED=6\sin 30°$

$\quad=3$

$AE=6\cos 30°=6\times\dfrac{\sqrt{3}}{2}=3\sqrt{3}$

$\because\ AB=AC$

$\therefore\ DB=3+3\sqrt{3}-6$

$\quad=3\sqrt{3}-3\ \ \text{(cm)}$

19. [D]

設 $AD=x$ cm，

則 $BD=42-x$ cm。

$\dfrac{36x}{2}-\dfrac{36(42-x)}{2}=216$

$x-(42-x)=12$

$x=27$

$AD=27$ ，$DB=42-27=15$

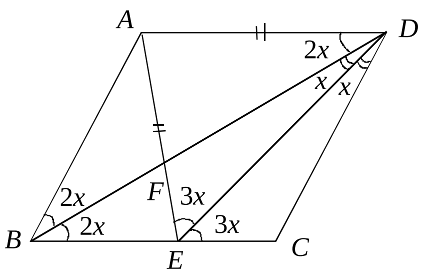
$AC=\sqrt{27^2+36^2}=45$

$BC=\sqrt{15^2+36^2}=39$

ΔABC 周界 $=42+39+45$

$=126\ \ \text{(cm)}$

20. [A]



$\because\ AE=AD$

$AD=DC$

$\therefore\ AE=DC$

故 I. 成立。

設 $\angle BDE=\angle EDC=x$ ，

由菱形性質及 $AE=AD$ ，可得

圖上各角的大小。

故 II. 成立。

$\Delta AFD\sim\Delta EFB$

$\therefore\ \dfrac{AF}{EF}=\dfrac{AD}{EB}\neq 1$

故 III. 不成立。

21. [C]

$\because$ AD 為直徑，

$\therefore$ $\angle ACD = 90°$

$\angle ADC = 180° - 122° = 58°$

$\angle BAC = \angle CAD = 90° - 58° = 32°$

$\angle ACB = 180° - 122° - 32° = 26°$

22. [B]

$AB = \dfrac{BC}{\cos(\alpha+\beta)}$ 。

$DC = BC\tan\alpha$

$$\frac{AB}{DC} = \frac{BC}{\cos(\alpha+\beta)} \times \frac{1}{BC\tan\alpha}$$

$$= \frac{1}{\tan\alpha\cos(\alpha+\beta)}$$

23. [B]

$ax + by = 6$

$m = -\dfrac{a}{b} > 0$ 。

$\therefore$ a 、b 異號。

當 $x = 0$ ，$y = \dfrac{6}{b} < 2$ ……(1)

當 $y = 0$ ，$x = \dfrac{6}{a} < -3$ ……(2)

$\therefore$ $a < 0$ ，$b > 0$

由 (1) 得 $b > 3$ 。

故 II. 不成立。 (由此已可知答案為 B)

由 (2) 得 $-3a < 6$ ($\because a < 0$)

$\therefore$ $a > -2$

故 I. 成立。

由 $a > -2$ 及 $b > 3$，得 $a + b > 0$ 。

故 III. 成立。

24. [A]

$L_1: 3x+4y-96=0$

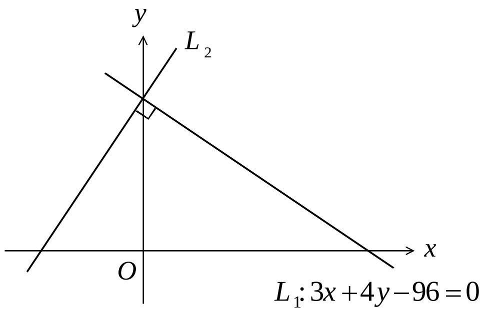
$x=0$ ， $y=24$ ； $y=0$ ， $x=32$ 。

$m_{L_1}=-\dfrac{3}{4}$ ， $m_{L_2}=\dfrac{4}{3}$

$L_2: y=\dfrac{4}{3}x+24$

$y=0$ ， $x=-18$

所求面積 $=\dfrac{(32+18)\times 24}{2}=600$



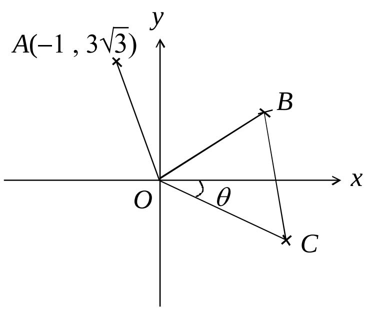
25. [D]

$B=(3\sqrt{3}\,,1)$ ， $C=(3\sqrt{3}\,,-3)$

$OC=\sqrt{(3\sqrt{3})^2+3^2}=6$

$\tan\theta=\dfrac{3}{3\sqrt{3}}=\dfrac{1}{\sqrt{3}}$ ， $\theta=30°$

$C=(6\,,330°)$



26. [D]

留意軌跡為兩條角平分線。

27. [C]

$C_1: x^2+y^2+16x+12y+25=0$

$X_1=(-8\,,-6)$ ， $r_1=\dfrac{1}{2}\sqrt{16^2+12^2-4\times 75}=5$

$C_2: 4x^2+4y^2-24x-32y+75=0$

$x^2+y^2-6x-8y+\dfrac{75}{4}=0$

$X_2=(3\,,4)$ ， $r_2=\dfrac{1}{2}\sqrt{6^2+8^2-75}=\dfrac{5}{2}$

$m_{ox_1}=\dfrac{0+6}{0+8}=\dfrac{3}{4}$ ， $m_{ox_2}=\dfrac{4-0}{3-0}=\dfrac{4}{3}$

$m_{ox_1}\neq m_{ox_2}$

故 I. 不成立。

$OX_1=r_1=2r_2=2OX_2$

故 II. 成立。

因為 II. 成立，由此知 III. 成立。

28. [C]

留意 2，3，5 及 7 為質數。

所求概率 $= \dfrac{C_2^4}{C_2^8} = \dfrac{3}{14}$

別解：

所求概率 $= \dfrac{6\times 2}{8\times 7} = \dfrac{3}{14}$

29. [A]

平均值及標準差不能從累積頻數多邊形中求出。

30. [B]

由於眾數為 23，故有至少 3 個數為 23。

不妨設 $p = q = 23$。

再由平均值為 22，得

$r = 22\times 10 - 213 = 7$

中位數 $= \dfrac{19+23}{2} = 21$

31. [D]

由 「左加右減」 法則知 I. 成立。

又 $y = \mathrm{f}(-x+11) = \mathrm{f}[-(x-11)] = \mathrm{g}(x)$

$\therefore$ III. 成立。

32. [A]

$32^3 + 32^9$

$= 2^{15} + 2^{45}$

$= 2^{12}\cdot 2^3 + 2^{44}\cdot 2$

$= 8(2^4)^3 + 2(2^4)^{11}$

$= 8\times 16^3 + 2\times 16^{11}$

$= 200000008000_{16}$

33. [C]

$$2\log x-\frac{3}{\log x}=5$$

$$2\log^2 x-5\log x-3=0$$

$$(2\log x+1)(\log x-3)=0$$

$$\log x=-\frac{1}{2}\ \text{或}\ \log x=3$$

$$\because\ \log\frac{1}{x}=-\log x$$

$$\therefore\ \log\frac{1}{x}=\frac{1}{2}\ \text{或}\ -3$$

34. [D]

$$\log_4 y=-\frac{2}{3}\log_2 x+2$$

$$\frac{\log_2 y}{2}=-\frac{2}{3}\log_2 x+2$$

$$\log_2 y=-\frac{4}{3}\log_2 x+4$$

$$\log_2 y=\log_2 x^{-\frac{4}{3}}+\log_2 16$$

$$y=16x^{-\frac{4}{3}}$$

$$y^3=16^3x^{-4}$$

$$x^4y^3=4096$$

35. [A]

$$z=\frac{3ai+6i^{10}}{1-i}$$

$$=\frac{3ai-6}{1-i}\times\frac{1+i}{1+i}\qquad(i^{10}=i^8\cdot i^2=-1)$$

$$=\frac{3ai-3a-6-6i}{2}$$

$$=\frac{-3a-6}{2}+\frac{3a-6}{2}i$$

由於 z 為純虛數，

故 $\frac{-3a-6}{2}=0$ ， $a=-2$

36. [B]

由 a 、 b 、 c 成一等差數列，

得 $c-b=b-a$

故 I. 不成立。

又 $\dfrac{3^c}{3^b}=3^{c-b}$ ，而 $\dfrac{3^b}{3^a}=3^{b-a}$

故 II. 成立。

III. 只有當 $n=1$ 時才成立。

37. [C]

連 QR ，並設 $\angle PRQ=x$ 。

$\because$ $QP=QR$

$\therefore$ $x=a$

又 $b+x=108°$

$\therefore$ $a+b=108°$

而 $a=\dfrac{180°-108°}{2}=36°$

$b=108°-36°=72°$

$\therefore$ $b-a=72°-36°=36°$

38. [C]

$\dfrac{360°}{k}=2\times(105°-45°)$

$k=3$

$y=\cos(3x°+\theta)=\cos 3(x°+\dfrac{\theta}{3})$

$\dfrac{\theta}{3}=-15°$ ， $\theta=-45°$

39. [B]

將 $y=3x-10$ 代入 $x^2+y^2=10$

得 $x^2+(3x-10)^2=10$

$$10x^2-60x+90=0$$

$$x^2-6x+9=0$$

$x=3$ (重根)

$y=-1$

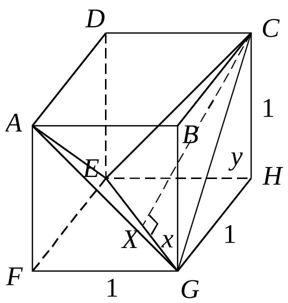
切點為 $(3,-1)$

由對稱性知另一切點為 $(-1,3)$

切線方程為 $\dfrac{y-5}{x-5}=\dfrac{5-3}{5-(-1)}$

即 $x-3y+10=0$

40. [D]



作 $CX\perp EG$，並設 $XG=x$。

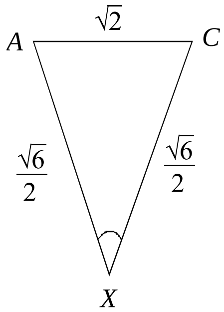
$$x^2+x^2=1$$

$$x=\frac{1}{\sqrt{2}}$$

$$y=\sqrt{2}$$

$$CX=\sqrt{2-\frac{1}{2}}=\frac{\sqrt{6}}{2}$$

設交角 $\angle AXC=\theta$。



$$\sin\frac{\theta}{2}=\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2}}=\frac{1}{\sqrt{3}}$$

$\theta=71°$ (準確至最接近的度)

41. [A]

I. 及 II. 顯然成立。

外心位於 ΔABC 外且面對 $\angle ABC$，而垂心位於 ΔABC 外且背對 $\angle ABC$。因此 III. 不成立。

42. [B]

考慮數字 0 位於個位及數字 5 位於個位兩種情況。

對後一種情況，要留意 0 不可以位於千位。

所求數目 $=P_3^5+C_1^4P_2^4$

$=60+48$

$=108$

43. [D]

所求概率 $= \frac{7}{9}\times\frac{6}{8}\times\frac{5}{7}+\frac{2}{9}\times\frac{7}{8}\times\frac{6}{7}\times 3$　　$\left(P=\frac{C_3^7+C_1^2C_2^7}{C_3^9}\right.$

$= \frac{11}{12}$　　$\left.=\frac{11}{12}\right)$

別解：

所求概率 $=1-\frac{2}{9}\times\frac{1}{8}\times\frac{7}{7}\times 3$　　$\left(P=1-\frac{C_2^2\times C_1^7}{C_3^9}\right.$

$= \frac{11}{12}$　　$\left.=\frac{11}{12}\right)$

44. [D]

設標準差為 s kg。

則 $\frac{54-50}{s}-\frac{48-50}{s}=1$

$\frac{6}{s}=1$

$s=6$

45. [A]

顯然 $m_1=m_2$，

$r_1=r_2$，

及 $v_1\geq v_2$

$\therefore$ 只有 A. 必然成立。

2019 / 20 MATHEMATICS Compulsory Part

Paper 2 Solutions

Answers (From left to right)

D A C D C   C B D A B   B A B C C   D A B D A
C B B A D   D C C A B   D A C D A   B C C B D
A B D D A

1. [D]

$$(\frac{9}{27^{n-1}})^4 = (\frac{3^2}{(3^3)^{n-1}})^4 = \frac{3^8}{(3^{3n-3})^4}$$

$$= \frac{3^8}{3^{12n-12}} = 3^{20-12n}$$

2. [A]

$$\begin{aligned} & m^2 + 2m - 4n^2 - 4n \\ &= m^2 - 4n^2 + 2m - 4n \\ &= (m+2n)(m-2n) + 2(m-2n) \\ &= (m-2n)(m+2n+2) \end{aligned}$$

3. [C]

$$\begin{cases} 3x - 2y = 11 & \cdots\cdots(1) \\ x + 4y - 2 = 11 & \cdots\cdots(2) \end{cases}$$

$(2)\times 3$, we have $3x + 12y = 39$ $\cdots\cdots(3)$

$(1)-(2)$, we have $-14y = -28$

$y = 2$

Sub. into (1) , we have $3x - 4 = 11$, $x = 5$.

$\therefore$ $x - y = 5 - 2 = 3$

4. [D]

$$(3x+4)(x-\alpha) \equiv 3x^2 + \beta(x+1)$$

$$3x^2 + (4-3\alpha)x - 4\alpha \equiv 3x^2 + \beta x + \beta$$

$$\begin{cases} \beta = 4 - 3\alpha \\ \beta = -4\alpha \end{cases}$$

$$4 - 3\alpha = -4\alpha$$

$$\alpha = -4 \quad , \quad \beta = 16$$

5. [C]

$$k=\frac{6}{5-h}-3$$

$$k+3=\frac{6}{5-h}$$

$$5-h=\frac{6}{k+3}$$

$$h=5-\frac{6}{k+3}$$

$$=\frac{5k+9}{k+3}$$

6. [C]

$$\sqrt{2019}+\sqrt{2020}\approx 89.87769399$$

$$=89.88 \quad \text{(correct to } \quad 4 \quad \text{significant figures)}$$

7. [B]

$$\frac{3-2x}{4}<-1 \quad \text{or} \quad 3x+7\geq 1$$

$$3-2x<-4 \quad \text{or} \quad 3x\geq -6$$

$$-2x<-7 \quad \text{or} \quad x\geq -2$$

$$x>\frac{7}{2} \quad \text{or} \quad x\geq -2$$

$\therefore \quad x\geq -2$

Answer : $x=-2$

8. [D]

$$\text{f}\,(x)=3x^2-4x-1$$

$$\text{f}\,(2c-1)=3(2x-1)^2-4(2c-1)-1$$

$$=3(4c^2-4c+1)-8c+4-1$$

$$=12c^2-20c+6$$

9. [A]

$$9x^2+(k+1)x+1=0$$

$$\Delta=(k+1)^2-36=0$$

$$k+1=\pm 6$$

$k=5$ (rejected) or $k=-7$

10. [B]

Vertex $=(p\ ,\ pq)$.

From the graph, both p and pq are negative.

$\therefore$ $p<0$ and $q>0$.

11. [B]

$$50000(1+\frac{x}{2}\%)^8=56325$$

$$(1+\frac{x}{200})^8=\frac{56325}{50000}$$

$$1+\frac{x}{200}=\sqrt[8]{\frac{56325}{50000}}$$

$$x=200(\sqrt[8]{\frac{56325}{50000}}-1)$$

$$=3$$

12. [A]

$$9k^2\ \text{cm}^2=22500\ \text{m}^2$$

$$9k^2\ \text{cm}^2=22500\times 100^2\ \text{cm}^2$$

$$9k^2=22500\times 100^2$$

$$k=5000$$

13. [B]

$z=\dfrac{k\sqrt{x}}{y^2}$, k is a constant

$$\frac{zy^2}{\sqrt{x}}=k$$

$$\frac{z^2y^4}{x}=k^2$$

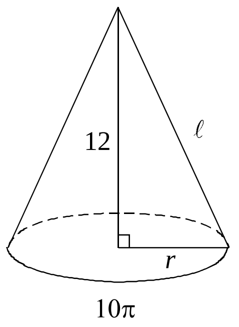
$$\frac{x}{y^4z^2}=\frac{1}{k^2}=\text{constant}$$

14. [C]

Number of dots $= 5+8+12+16+20+24+28+32$

$= 145$

15. [C]



$2\pi r = 10\pi$

$r = 5$

$\ell = \sqrt{5^5 + 12^2} = 13$

Total surface area

$= \pi \times 5^2 + \pi \times 5 \times 13$

$= 90\pi$ (cm^2)

16. [D]

$\Delta BCE \sim \Delta FDE$

The area of ΔBCE : The area of ΔDEF $= 2^2 : 3^2 = 4:9$

Let x cm^2 be the area of ΔBCE,

then the area of ΔDEF $= \frac{9}{4}x$ cm^2.

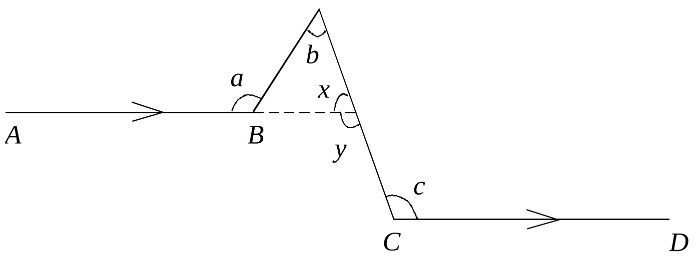
$\therefore \quad x + \frac{9}{4}x = 78$

$\frac{13}{4}x = 78$

$x = 24$

The area of ΔABE $= 24 \times \frac{5}{2} = 60$ (cm^2)

17. [A]



$x = a - b$

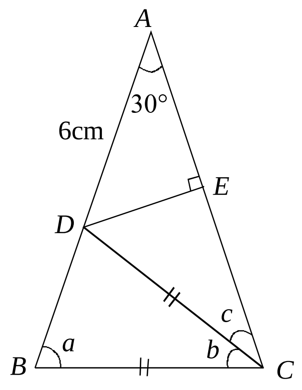
$y = 180° - x = 180° - a + b$

$c = y = 180° - a + b$

$\therefore \quad a - b + c = 180°$

18. [B]

$a = \dfrac{180° - 30°}{2} = 75°$



$b = 180° - 2 \times 75° = 30°$

$c = 75° - 30° = 45°$

$EC = ED = 6\sin 30°$

$= 3$

$AE = 6\cos 30° = 6 \times \dfrac{\sqrt{3}}{2} = 3\sqrt{3}$

$\because \quad AB = AC$

$\therefore \quad DB = 3 + 3\sqrt{3} - 6$

$= 3\sqrt{3} - 3 \quad$ (cm)

19. [D]

Let $AD = x$ cm,

then $BD = 42 - x$ cm.

$\dfrac{36x}{2} - \dfrac{36(42-x)}{2} = 216$

$x - (42 - x) = 12$

$x = 27$

$AD = 27$, $DB = 42 - 27 = 15$

$AC = \sqrt{27^2 + 36^2} = 45$

$BC = \sqrt{15^2 + 36^2} = 39$

The perimeter of $\Delta ABC = 42 + 39 + 45$

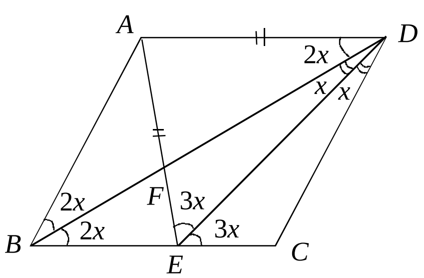
$= 126$ (cm)

20. [A]

$\because \quad AE = AD$

$AD = DC$

$\therefore \quad AE = DC$



Thus I. is true .

Let $\angle BDE = \angle EDC = x$,

from the properties of rhombus and $AE = AD$, we get the sizes of all angles in the figure .

Thus II. is true .

$\Delta AFD \sim \Delta EFB$

$\therefore \quad \dfrac{AF}{EF} = \dfrac{AD}{EB} \neq 1$

Thus, III. is false .

21. [C]

$\because$ AD is a diameter ,

$\therefore$ $\angle ACD = 90°$

$\angle ADC = 180° - 122° = 58°$

$\angle BAC = \angle CAD = 90° - 58° = 32°$

$\angle ACB = 180° - 122° - 32° = 26°$

22. [B]

$AB = \dfrac{BC}{\cos(\alpha+\beta)}$.

$DC = BC\tan\alpha$

$\dfrac{AB}{DC} = \dfrac{BC}{\cos(\alpha+\beta)} \times \dfrac{1}{BC\tan\alpha}$

$= \dfrac{1}{\tan\alpha\cos(\alpha+\beta)}$

23. [B]

$ax + by = 6$

$m = -\dfrac{a}{b} > 0$.

$\therefore$ a and b are of different signs.

When $x = 0$, $y = \dfrac{6}{b} < 2$ $\cdots\cdots(1)$

When $y = 0$, $x = \dfrac{6}{a} < -3$ $\cdots\cdots(2)$

$\therefore$ $a < 0$, $b > 0$

From (1) , we have $b > 3$.

Thus, II. is false . (Hence, the answer is B)

From (2) , we have $-3a < 6$ ($\because$ $a < 0$)

$\therefore$ $a > -2$

Thus, I. is true .

From $a > -2$ and $b > 3$, we have $a + b > 0$.

Thus, III. is true .

24. [A]

$L_1 : 3x + 4y - 96 = 0$

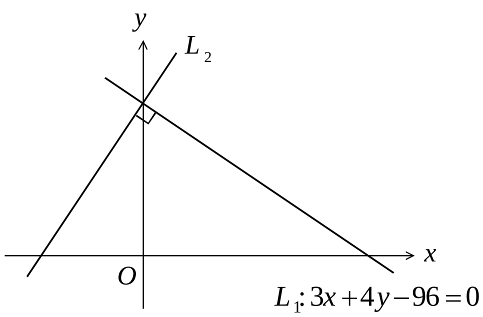
$x = 0$, $y = 24$; $y = 0$, $x = 32$.

$m_{L_1} = -\dfrac{3}{4}$, $m_{L_2} = \dfrac{4}{3}$

$L_2 : y = \dfrac{4}{3}x + 24$

$y = 0$ ， $x = -18$

The required area $= \dfrac{(32+18)\times 24}{2} = 600$



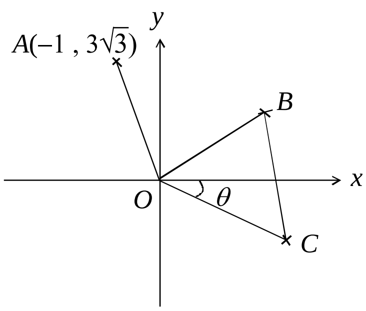
25. [D]

$B = (3\sqrt{3}\,, 1)$ ， $C = (3\sqrt{3}\,, -3)$

$OC = \sqrt{(3\sqrt{3})^2 + 3^2} = 6$

$\tan\theta = \dfrac{3}{3\sqrt{3}} = \dfrac{1}{\sqrt{3}}$, $\theta = 30°$

$C = (6\,, 330°)$



26. [D]

Note that the locus is a pair of angle bisectors .

27. [C]

$C_1 : x^2 + y^2 + 16x + 12y + 25 = 0$

$X_1 = (-8\,, -6)$ ， $r_1 = \dfrac{1}{2}\sqrt{16^2 + 12^2 - 4\times 75} = 5$

$C_2 : 4x^2 + 4y^2 - 24x - 32y + 75 = 0$

$x^2 + y^2 - 6x - 8y + \dfrac{75}{4} = 0$

$X_2 = (3\,, 4)$ ， $r_2 = \dfrac{1}{2}\sqrt{6^2 + 8^2 - 75} = \dfrac{5}{2}$

$m_{ox_1} = \dfrac{0+6}{0+8} = \dfrac{3}{4}$ ， $m_{ox_2} = \dfrac{4-0}{3-0} = \dfrac{4}{3}$

$m_{ox_1} \neq m_{ox_2}$

Thus, I. is false .

$OX_1 = r_1 = 2r_2 = 2OX_2$

Thus II. is true .

Since II. is true , III. is also true .

28. [C]

Note that 2 , 3 , 5 and 7 are prime numbers.

The required probability $=\dfrac{C_2^4}{C_2^8}=\dfrac{3}{14}$

Alternative solution:

The required probability $=\dfrac{6\times 2}{8\times 7}=\dfrac{3}{14}$

29. [A]

Mean and standard deviation cannot be obtained from a cumulative frequency polygon .

30. [B]

Since the mode is 23, at least three numbers are 23 .

Suppose that $p=q=23$.

Since the mean is 22 , we have

$r=22\times 10-213=7$

The median $=\dfrac{19+23}{2}=21$

31. [D]

The graph of $g(x)$ is obtained by translating the graph of $f(x)$ to the right, I. is true.

And $y=f(-x+11)=f[-(x-11)]=g(x)$

$\therefore$ III. is true.

32. [A]

32^3+32^9

$=2^{15}+2^{45}$

$=2^{12}\cdot 2^3+2^{44}\cdot 2$

$=8(2^4)^3+2(2^4)^{11}$

$=8\times 16^3+2\times 16^{11}$

$=200000008000_{16}$

33. [C]

$$2\log x - \frac{3}{\log x} = 5$$

$$2\log^2 x - 5\log x - 3 = 0$$

$$(2\log x + 1)(\log x - 3) = 0$$

$$\log x = -\frac{1}{2} \quad \text{or} \quad \log x = 3$$

$$\because \quad \log\frac{1}{x} = -\log x$$

$$\therefore \quad \log\frac{1}{x} = \frac{1}{2} \quad \text{or} \quad -3$$

34. [D]

$$\log_4 y = -\frac{2}{3}\log_2 x + 2$$

$$\frac{\log_2 y}{2} = -\frac{2}{3}\log_2 x + 2$$

$$\log_2 y = -\frac{4}{3}\log_2 x + 4$$

$$\log_2 y = \log_2 x^{-\frac{4}{3}} + \log_2 16$$

$$y = 16x^{-\frac{4}{3}}$$

$$y^3 = 16^3 x^{-4}$$

$$x^4 y^3 = 4096$$

35. [A]

$$z = \frac{3ai + 6i^{10}}{1-i}$$

$$= \frac{3ai - 6}{1-i} \times \frac{1+i}{1+i} \qquad (\, i^{10} = i^8 \cdot i^2 = -1\,)$$

$$= \frac{3ai - 3a - 6 - 6i}{2}$$

$$= \frac{-3a-6}{2} + \frac{3a-6}{2}i$$

Since z is a pure imaginary number ,

so, $\frac{-3a-6}{2} = 0$, $a = -2$

36. [B]

Since a , b , c is an arithmetic sequence ,

we have $c-b=b-a$

Thus, I. is false .

$\frac{3^c}{3^b}=3^{c-b}$ and $\frac{3^b}{3^a}=3^{b-a}$

Thus, II. is true .

III. is true only when $n=1$.

37. [C]

Join QR , and let $\angle PRQ = x$.

$\because$ $QP = QR$

$\therefore$ $x = a$

and $b + x = 108°$

$\therefore$ $a + b = 108°$

and $a = \frac{180° - 108°}{2} = 36°$

$b = 108° - 36° = 72°$

$\therefore$ $b - a = 72° - 36° = 36°$

38. [C]

$\frac{360°}{k} = 2 \times (105° - 45°)$

$k = 3$

$y = \cos(3x° + \theta) = \cos 3(x° + \frac{\theta}{3})$

$\frac{\theta}{3} = -15°$, $\theta = -45°$

39. [B]

Sub. $y=3x-10$ into $x^2+y^2=10$,

we have $x^2+(3x-10)^2=10$

$$10x^2-60x+90=0$$

$$x^2-6x+9=0$$

$$x=3 \quad \text{(repeated)}$$

$$y=-1$$

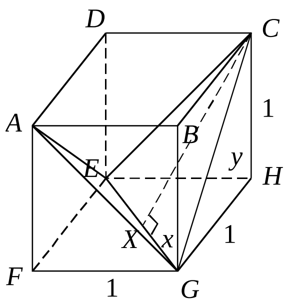
The point of contact is $(3,-1)$

From symmetry, we know that $(-1,3)$ is another point of contact

The equation of tangent is $\dfrac{y-5}{x-5}=\dfrac{5-3}{5-(-1)}$

i.e. $x-3y+10=0$

40. [D]



Construct $CX\perp EG$ and let $XG=x$.

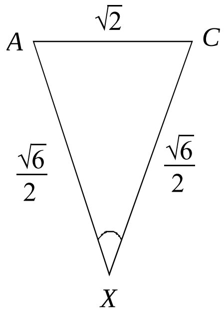
$$x^2+x^2=1$$

$$x=\frac{1}{\sqrt{2}}$$

$$y=\sqrt{2}$$

$$CX=\sqrt{2-\frac{1}{2}}=\frac{\sqrt{6}}{2}$$

Let $\angle AXC=\theta$ be the required angle .



$$\sin\frac{\theta}{2}=\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2}}=\frac{1}{\sqrt{3}}$$

$$\theta=71^\circ \quad \text{(correct to the nearest degree)}$$

41. [A]

Obviously, I. and II. are true .

Circumcentre lies outside ΔABC and faces $\angle ABC$, while orthocentre lies outside ΔABC and is opposite to $\angle ABC$. Thus, III. is false .

42. [B]

Consider the two situations where 0 or 5 being the units digit respectively .

Note that 0 cannot be put at thousands when the units digit is 5 .

The required numbers $=P_3^5+C_1^4P_2^4$

$=60+\;4$

$=108$

43. [D]

The required probability $= \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} + \frac{2}{9} \times \frac{7}{8} \times \frac{6}{7} \times 3$ $\quad (P = \frac{C_3^7 + C_1^2 C_2^7}{C_3^9}$

$= \frac{11}{12}$ $\quad = \frac{11}{12})$

Alternative solution :

The required probability $= 1 - \frac{2}{9} \times \frac{1}{8} \times \frac{7}{7} \times 3$ $\quad (P = 1 - \frac{C_2^2 \times C_1^7}{C_3^9}$

$= \frac{11}{12}$ $\quad = \frac{11}{12})$

44. [D]

Let s kg be the standard deviation ,

then $\frac{54-50}{s} - \frac{48-50}{s} = 1$

$\frac{6}{s} = 1$

$s = 6$

45. [A]

Obviously $m_1 = m_2$,

$r_1 = r_2$,

and $v_1 \geq v_2$

$\therefore$ Only A. must be true.