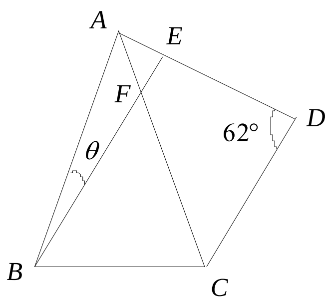
Paper I Answers (2020/21)

Solution	Marks	Remarks
1. $\dfrac{(a^3b^{-2})^4}{a^{-5}}$		
$=\dfrac{a^{12}b^{-8}}{a^{-5}}$	1M	for $(xy)^m = x^m y^m$ or $(x^m)^n = x^{mn}$
$=\dfrac{a^{12-(-5)}}{b^8}$	1M	for $z^{-p}=\dfrac{1}{z^p}$ or $\dfrac{z^p}{z^q}=z^{p-q}$
$=\dfrac{a^{17}}{b^8}$	1A	
	--------(3)	
2. (a) $x^2-6xy+9y^2$		
$=(x-3y)^2$	1A	or $(3y-x)^2$
(b) $x^2-6xy+9y^2-4$		
$=(x-3y)^2-4$	1M	for using the result of (a)
$=(x-3y+2)(x-3y-2)$	1A	or equivalent
	--------(3)	
3. $5a=3b$		
$a:b=3:5=6:10$	1A	either one
$c=\dfrac{b}{2}$		
$b:c=2:1=10:5$		
$\therefore\ a:b:c=6:10:5$		
Let $a=6k$, $b=10k$, $c=5k$, where $k\neq 0$		
By substitution, $2a+b-3c=14$		
We have $12k+10k-15k=14$	1M	
$7k=14$		
$k=2$		
$\therefore\ c=5k=10$	1A	
	--------(3)	
4. The selling price $=\$160\times(1+10\%)$	1M	
$=\$176$	1A	
The marked price $=\$176\div 80\%$	1M	
$=\$220$	1A	
	--------(4)	

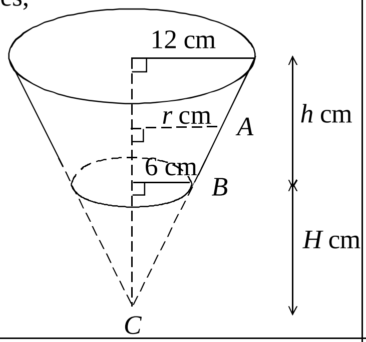
	Solution	Marks	Remarks
5.	Let x be the number of female members,		
	then the number of male members is $\frac{4}{3}x$.	1A	
	$x+\frac{4}{3}x=280$	1M+1A	1M for getting a linear equation in one unknown
	$\frac{7}{3}x=280$		
	$x=120$		
	The difference between the number of male members and the number of female members $=\frac{1}{3}\times 120$		
	$=40$	1A	
	The difference between the number of male members and the number of female members $=280-2\times 120$		
	$=40$	1A	
	Let x and y be the number of male members and the number of female members respectively.		
	Then $x=\frac{4}{3}y$ and $x+y=280$	1A+1A	
	$\therefore\ \frac{4}{3}y+y=280$	1M	1M for getting a linear equation in one unknown
	$\frac{7}{3}y=280$		
	$y=120$		
	$x=160$		
	The difference between the number of male members and the number of female members $=160-120$		
	$=40$	1A	
		--------(4)	
6.	(a) $6-x>\frac{3-4x}{2}$		
	$12-2x>3-4x$ $\quad (6-x>\frac{3}{2}-2x)$		
	$-2x+4x>3-12$ $\quad (-x+2x>\frac{3}{2}-6)$	1M	for putting x on one side
	$2x>-9$		
	$\therefore\ x>-\frac{9}{2}$	1A	
	$42-7x\le 0$		
	$7x\ge 42$		
	$x\ge 6$		
	$\therefore$ The solution of the compound inequality is $x>-\frac{9}{2}$.	1A	
	(b) 5	1A	
		--------(4)	

Solution	Marks	Remarks
7. (a) The coordinates of A' are $(7, 1)$.	1A	accept $A'(7, 1)$ or $A' = (7, 1)$
The coordinates of B' are $(-4, -4)$.	1A	accept $B'(-4, -4)$ or $B' = (-4, -4)$
(b) The slope of $AB = \frac{7+4}{-1-4} = -\frac{11}{5}$	1M	accept $m_{AB} = -\frac{11}{5}$ either one
The slope of $A'B' = \frac{1+4}{7+4} = \frac{5}{11}$		
$\because$ The slope of AB $\times$ The slope of $A'B'$ $= (-\frac{11}{5})(\frac{5}{11})$ $= -1$		
$\therefore$ $AB \perp A'B'$	1	f.t.
	--------(4)	
8. $\because$ $AC = AD$		
$\therefore$ $\angle ACD = \angle ADC$ $= 62°$	1M	
$\because$ $BE // CD$		
$\therefore$ $\angle AFE = \angle ACD$ $= 62°$	1M	
$\angle BAC = \angle AFE - \theta$ $= 62° - \theta$	1A	
$\because$ $AB = AC$		
$\therefore$ $\angle ABC = \frac{180° - \angle BAC}{2}$	1M	
$= \frac{180° - (62° - \theta)}{2}$ $= 59° + \frac{\theta}{2}$		
$\angle FBC = 59° + \frac{\theta}{2} - \theta$		
$= 59° - \frac{\theta}{2}$	1A	
	--------(5)	

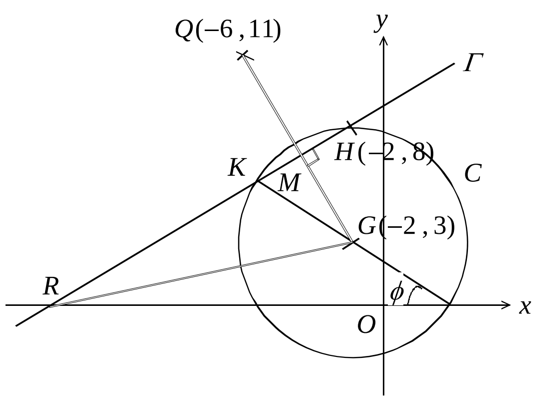


Solution	Marks	Remarks
9. (a) $\dfrac{b}{19+a+b}=\dfrac{1}{8}$	1M	
$8b=19+a+b$		
$a-7b=-19 \quad ---(1)$		
and $7+a=12+b$	1M	
$a-b=5 \quad ---(2)$		
By solving (1) and (2), we have $a=9$, $b=4$.	1A	for both correct
(b) The original mean $=6.40625$	1M	either one
New mean $=6.375$		
The decrease in the mean is 0.03125	1A	r.t. 0.0313
The decrease in the mean is $\dfrac{1}{32}$ $=0.03125$	1M+1A	1M for numerator
	--------(5)	

		Solution	Marks	Remarks
10.	(a)	$S = aA + bA^2$, where a and b are non-zero constants.	1A	
		Sub. $A = 4$, $S = 56$ and $A = 7$, $S = 140$		
		We have $4a + 16b = 56$	1M	for either substitution
		$a + 4b = 14 \quad ---(1)$		
		and $7a + 49b = 140$		
		$a + 7b = 20 \quad ---(2)$		
		By solving (1) and (2), we have $a = 6$, $b = 2$.	1A	for both correct
		$\therefore \quad S = 6A + 2A^2$		
		When $A = 6$, $S = 6 \times 6 + 2 \times 6^2$		
		$= \$108$	1A	
			--------(4)	
	(b)	When $A = 12$, $S = 6 \times 12 + 2 \times 12^2$	1M	
		$= \$360$		
		$\neq 4 \times \$108$		
		$= \$432$		
		$\therefore$ The claim is not correct.	1A	f.t.
			--------(2)	
11.	(a)	The inter-quartile range		
		$= 128 - 114$	1M	
		$= 14$(s)	1A	
			--------(2)	
	(b)	(i) $130 + b - (100 + a) \geq 14 + 24$	1M	
		$b - a \geq 8$		
		$\begin{cases} a = 0 \\ b = 8 \end{cases}$ or $\begin{cases} a = 0 \\ b = 9 \end{cases}$ or $\begin{cases} a = 1 \\ b = 9 \end{cases}$	1A	for at least two correct.
		(ii) When $a = 0$ and $b = 9$, the standard deviation is the greatest.	1M	
		The greatest possible standard deviation ≈ 9.20271699		
		≈ 9.20(s)	1A	r.t. 9.20(s)
		When $a = 0$ and $b = 8$,		
		the standard deviation $\approx 9.107551812 \approx 9.11$(s)		
		When $a = 0$ and $b = 9$,		
		the standard deviation $\approx 9.20271699 \approx 9.20$(s)		
		When $a = 1$ and $b = 9$,		
		the standard deviation $\approx 9.089966997 \approx 9.09$(s)		
		Since 9.20 > 9.11 > 9.09	1M	
		$\therefore$ The greatest possible standard deviation ≈ 9.20(s)	1A	r.t. 9.20(s)
			--------(4)	

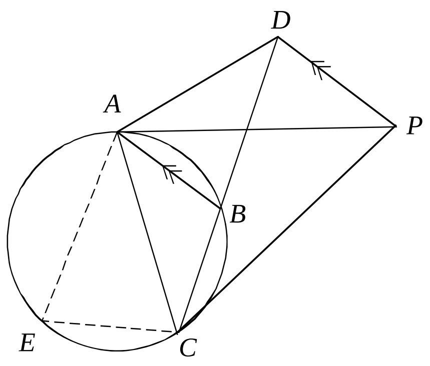
		Solution	Marks	Remarks
12.	(a)	By using the similar triangles,		
		we have $\dfrac{H}{h+H}=\dfrac{6}{12}$	1M	
		$H=h$		
				
		$\dfrac{1}{3}\pi\times12^2\times2h-\dfrac{1}{3}\pi\times6^2h=672\pi$	1M	
		$96h-12h=672$		
		$84h=672$		
		$h=8$	1A	
		$\dfrac{1}{3}\pi\times6^2h=672\pi\times\dfrac{1^3}{2^3-1^3}$	1M	
		$12h=96$		
		$h=8$	1A	
			--------(3)	
	(b)	Let r cm be the radius of the water surface.		
		By using the similar triangles, we have		
		$\dfrac{6}{r}=\dfrac{8}{12}$	1M	
		$r=9$		
		$AC=\sqrt{9^2+12^2}$	1M	either one
		$=15$		
		$BC=\sqrt{6^2+8^2}$		
		$=10$		
		The area of the vessel wetted by the water		
		$=\pi\cdot9\cdot15-\pi\cdot6\cdot10+\pi\times6^2$	1M	1M for $\pi\cdot9\cdot15-\pi\cdot6\cdot10$
		$=111\pi\ (\text{cm}^2)$	1A	
		The area of the vessel wetted by the water		
		$=\pi\cdot9\cdot15\times\dfrac{15^2-10^2}{15^2}+\pi\times6^2$	1M	1M for $\pi\cdot9\cdot15\times\dfrac{15^2-10^2}{15^2}$
		$=111\pi\ (\text{cm}^2)$	1A	
			--------(4)	

		Solution	Marks	Remarks
13.	(a)	Let $f(x)=(x^2-1)(ax+b)+4x+k$, where a and b are constants.	1M	$f(x)=(x^2-1)q(x)+4x+k$,
		$f(-1)=-4+k=0$	1M	where $q(x)$ is a polynomial.
		$k=4$	1A	
			--------(3)	
	(b)	$f(x)=(x^2-1)(ax+b)+4x+4$		
		Since $f(0)=-4$	1M	
		$\therefore\ -b+4=-4$		
		$b=8$	1A	
		$\therefore\ f(x)=(x+1)(x-1)(ax+8)+4(x+1)$		
		$f(x)=0$		
		$(x+1)[(x-1)(ax+8)+4]=0$		
		If $f(x)=0$ has repeated roots different from -1 ,		
		then $(x-1)(ax+8)+4=0$ has repeated roots,	1M	
		$ax^2+(8-a)x-4=0$		
		$\Delta=(8-a)^2+16a=0$	1M	
		$a^2+64=0$		
		Since a is a real number,		
		the equation $(x-1)(ax+8)+4=0$ cannot have repeated roots.		
		i.e. the equation $f(x)=0$ cannot have repeated roots different from -1 .		
		Thus, the claim is correct.	1A	f.t.
			--------(5)	

Solution	Marks	Remarks
14. (a) $C: x^2+y^2+4x-6y-12=0$		
Sub. $(-2, b)$ into C, we have		
$4+b^2-8-6b-12=0$		
$b^2-6b-16=0$		
$(b+2)(b-8)=0$		
$b=-2$ (rejected) or $b=8$	1A	
	--------(1)	
(b) (i) The coordinates of G are $(-2, 3)$		
Let (x, y) be the coordinates of P		
Since $PQ=PG$, we have		
$\sqrt{(x+6)^2+(y-11)^2}=\sqrt{(x+2)^2+(y-3)^2}$	1M	
$8x-16y+144=0$		
$\Gamma: x-2y+18=0$	1A	or equivalent
The coordinates of M, the mid-point of QG are $(-4, 7)$		
The slope of QG $=\frac{11-3}{-6+2}=-2$		
The slope of Γ $=\frac{1}{2}$		
$\Gamma: \frac{y-7}{x+4}=\frac{1}{2}$	1M	
$x-2y+18=0$	1A	or equivalent
$\because$ $-2-2(8)+18=0$		
$\therefore$ Γ passes through H.	1A	f.t.
(ii) 		
Note that M is the mid-point of both QG and HK.		
Let (c, d) be the coordinates of K		
then $\frac{c-2}{2}=-4$, $\frac{d+8}{2}=7$		
$c=-6$, $d=6$	1A	for both correct

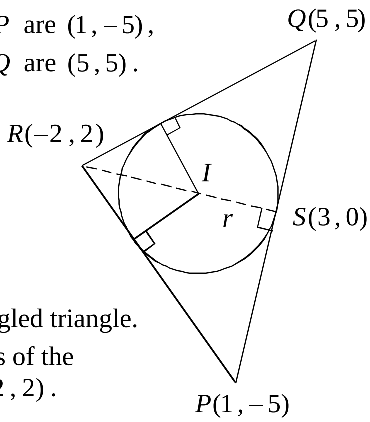
Solution	Marks	Remarks
The slope of $\Gamma = \frac{1}{2}$		
$\tan \angle KRO = \frac{1}{2}$ $\angle KRO \approx 26.56505118°$	1M	
The slope of the straight line $KG = \frac{6-3}{-6+2} = -\frac{3}{4}$ $\phi \approx 36.869897565°$		
$\because \angle KRG < 26.56505118°$ and $\angle KGR > 36.869897565°$ $\therefore \angle KGR > \angle KRG$	1M	either one
Thus, the claim is agreed.	1A	f.t.
The coordinates of R are $(-18, 0)$		
$KR = \sqrt{12^2 + 6^2} = \sqrt{180}$ $KG = \sqrt{4^2 + 3^2} = 5$	1M	
$\therefore KR > KG$ $\therefore \angle KGR > \angle KRG$	1M	$\angle KRG \approx 15.9453959°$ $\angle KGR \approx 47.48955292°$
Thus, the claim is agreed.	1A	f.t.
	--------(7)	

Solution	Marks	Remarks
15. (a) The required probability $=\dfrac{C_6^7+C_5^7C_1^5+C_4^7C_2^5}{C_6^{12}}$	1M	1M for numerator
$=\dfrac{1}{2}$	1A	
The required probability		
$=(\frac{7}{12})(\frac{6}{11})(\frac{5}{10})(\frac{4}{9})(\frac{3}{8})(\frac{2}{7})+(\frac{7}{12})(\frac{6}{11})(\frac{5}{10})(\frac{4}{9})(\frac{3}{8})(\frac{5}{7})\cdot C_1^6$	1M	1M for $P_1+P_2+P_3$
$+(\frac{7}{12})(\frac{6}{11})(\frac{5}{10})(\frac{4}{9})(\frac{5}{8})(\frac{4}{7})\cdot C_2^6$		
$=\dfrac{1}{2}$	1A	accept 0.5
	--------(2)	
(b) The required probability $=\dfrac{P_3^3\cdot P_3^4}{P_6^6}$	1M+1M	1M for numerator, 1M for denominator
$=\dfrac{1}{5}$	1A	
The required probability $=\dfrac{P_3^3\cdot C_3^4\cdot P_3^3}{P_6^6}$	1M+1M	1M for numerator, 1M for denominator
$=\dfrac{1}{5}$	1A	accept 0.2
	--------(3)	
16. (a) Let a be the 1st term of the sequence, r be the common ratio. Then		
$ar=200 \quad ---(1)$	1M	either one
$\dfrac{a}{1-r}=800 \quad ---(2)$		
$\dfrac{(2)}{(1)}$ we have $\dfrac{1}{r(1-r)}=4$		
$4r^2-4r+1=0$		
$\therefore\ r=\dfrac{1}{2},\ a=400$	1A	for both correct
	--------(2)	
(b) $\dfrac{400[1-(\frac{1}{2})^n]}{1-\frac{1}{2}}>800(1-10^{-10})$	1M	
$1-(\frac{1}{2})^n>1-10^{-10}$		
$(\frac{1}{2})^n<10^{-10}$		
$\log(\frac{1}{2})^n<\log 10^{-10}$	1M	
$n\log\frac{1}{2}<-10$		
$n>33.21928095$		
$\therefore$ The least value of n is 34.	1A	
	--------(3)	

Solution	Marks	Remarks
17. 		
(a) $\because$ $AB // DP$ (given) $\therefore$ $\angle DPA = \angle BAP$ (alt. $\angle$s) and $\angle BAP = \angle ACB$ ($\angle$ in alt. segment) $\therefore$ $\angle DPA = \angle ACB$ ($\angle ACD$) $\therefore$ A, C, P, D are concyclic. (converse of $\angle$s in the same segment)		accept $ACPD$ is a cyclic quadrilateral
Marking Scheme：		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	2	
Case 3 Incomplete proof with any one correct step and one correct reason.	1	
	--------(3)	
(Students can follow the following procedures to prove four points are concyclic) $\angle CDP = \angle ABD = \angle AEC = \angle CAP$)		
(b) $\because$ $ACPD$ is a cyclic quadrilateral		
$\therefore$ $\angle PAC = \angle PDC$	1M	
$= \angle DBA$ ($\because$ $AB // DP$)		
and $\angle DCA = \angle PAB$		
$\therefore$ $\angle PCD + \angle DCA = \angle PAD + \angle PAB$		
i.e. $\angle PCA = \angle DAB$	1A	
$\therefore$ $\Delta PAC \sim \Delta DBA$		accept students who prove $\Delta PAC \sim \Delta DAB$
Thus, the claim is agreed.	1A	f.t.
PD is produced to Q. $\angle PCA = \angle QDA$ $= \angle DAB$	1A	
	--------(3)	

Solution	Marks	Remarks
18. (a) $CB = \frac{3}{2}x$		
$FB = x + \frac{3}{2}x = \frac{5}{2}x$		
$\frac{1}{2}(\frac{5}{2}x)^2 - \frac{1}{2}x^2 = 42$	1M	
$x^2 = 16$		
$x = 4$	1A	
$DC = \sqrt{2}x$, $EB = \frac{5\sqrt{2}}{2}x$		
$\frac{(\sqrt{2}x + \frac{5\sqrt{2}}{2}x)\cdot\frac{3\sqrt{2}}{4}x}{2} = 42$	1M	
$x^2 = 16$		
$x = 4$	1A	
	--------(2)	
(b) (i) $A'E = AE = 10$, $DE = 6$		
In $\Delta A'DE$,		
$(A'D)^2 = 10^2 + 6^2 - 2(10)(6)\cos 40°$	1M	
$A'D \approx 6.638875419$		
≈ 6.64 (cm)	1A	
(ii) Construct $A'M \perp EB$ such that M is the foot of perpendicular, and also construct $A'N \perp DC$ such that N is the foot of perpendicular.		
The required angle is $\angle A'MN$ (Denote by θ).	1M	
$A'M = 10\sin 45° = 5\sqrt{2}$		
$DC = 4\sqrt{2}$, $DN = 2\sqrt{2}$		
$(A'N)^2 \approx 6.638875419^2 - (2\sqrt{2})^2$		
$A'N \approx 6.006219012$		
$MN \approx 6\sin 45° = 3\sqrt{2}$		
In $\Delta A'MN$,		
$\cos\theta \approx \frac{(5\sqrt{2})^2 + (3\sqrt{2})^2 - 6.006219012^2}{2(5\sqrt{2})(3\sqrt{2})}$	1M	
$\theta \approx 57.85329859°$		
$> 40°$		
$\therefore$ The claim is agreed.	1A	f.t.
	--------(5)	

Solution	Marks	Remarks
19. (a) $g(x)=\frac{1}{k}x^2-2x+3k-1$		
$=\frac{1}{k}(x^2-2kx)+3k-1$	1A	
$=\frac{1}{k}(x^2-2kx+k^2-k^2)+3k-1$	1M	
$=\frac{1}{k}(x-k)^2+2k-1$		
The coordinates of the vertex of the graph of $y=g(x)$ are $(k\,,2k-1)$.	1A	
	--------(3)	
(b) (i) $y=-g(x+2)$ The coordinates of P are $(k-2\,,1-2k)$ $y=g(8-x)$ The coordinates of Q are $(8-k\,,2k-1)$	1A	either one
Denote the circumcentre by $S(3\,,0)$		
Since $PS=RS$, we have		
$\sqrt{(k-5)^2+(1-2k)^2}=\sqrt{5^2+2^2}$	1M	
$5k^2-14k-3=0$		
$(k-3)(5k+1)=0$		
$\therefore\ k=3$ or $k=-\frac{1}{5}$ (rejected)	1A	
(ii) When $k=3$, the coordinates of P are $(1\,,-5)$, the coordinates of Q are $(5\,,5)$.		
$m_{QR}=\frac{3}{7}$ $m_{PR}=-\frac{7}{3}$ $\because\ m_{QR}\cdot m_{PR}=-1$ $\therefore\ \angle QRP=90°$	1M	consider one case
ΔPQR is a right-angled triangle. Thus, the coordinates of the orthocentre are $R(-2\,,2)$.	1A	
$m_{PQ}=\frac{5}{2}$, $m_{RQ}=\frac{3}{7}$ $(m_{PR}=-\frac{7}{3})$		
The equation of the altitude on PQ from R is $\frac{y-2}{x+2}=-\frac{2}{5}$ i.e. $2x+5y-6=0$ ---(1) The equation of the altitude on RQ from P is $\frac{y+5}{x-1}=-\frac{7}{3}$ i.e. $7x+3y+8=0$ ---(2)	1M	either one The equation of the altitude on RP from Q is $3x-7y+20=0$
By solving (1) and (2), the coordinates of the orthocentre are $R(-2\,,2)$.	1A	



Solution	Marks	Remarks
(iii) Since ΔPQR is a right-angled triangle, circumcentre $S(3, 0)$ is lying on the mid-point of the hypotenuse PQ.	1M	
Note that $PR = RQ = \sqrt{58}$, PQR is a right-angled isosceles triangle. The in-centre I is lying on the altitude RS.	1M	
Let r be the radius of the inscribed circle, then $RI = \sqrt{2}r$	1A	
$\therefore$ The radius of the circumcircle $= RS = (1+\sqrt{2})r$ Thus, the claim of the student is agreed.	1A	f.t.
The lengths of the three sides of the triangle PQR are $\sqrt{58}$, $\sqrt{58}$ and $2\sqrt{29}$. Let r be the radius of the inscribed circle, then $\frac{(\sqrt{58}+\sqrt{58}+2\sqrt{29})r}{2} = \frac{\sqrt{58}\cdot\sqrt{58}}{2}$	1M	
$r = \frac{29}{\sqrt{58}+\sqrt{29}}$	1A	
The radius of the circumcircle $= RS$ $= \sqrt{29}$ $\because \frac{RS}{r} = \frac{\sqrt{29}}{\frac{29}{\sqrt{58}+\sqrt{29}}}$ $= 1+\sqrt{2}$ $\therefore$ The claim of the student is agreed.	1A	f.t.
	--------(9)	

2020 / 21 MATHEMATICS Compulsory Part

Paper 2 Solutions

Answers (From left to right)

B C D B C   A A D D B   C C A D C   D C A A A
B C B C D   B A D B D   C A D B B   A C D A D
B B C A D

1. [B]

$$\frac{(6x^{-5})^{-2}}{4x}=\frac{6^{-2}x^{10}}{4x}$$
$$=\frac{x^9}{36\times 4}$$
$$=\frac{x^9}{144}$$

2. [C]

$$\frac{3a+b}{3a}=2-\frac{b}{a}$$
$$3a+b=6a-3b$$
$$4b=3a$$
$$b=\frac{3a}{4}$$

3. [D]

$$\frac{1}{5+3x}-\frac{1}{5-3x}$$
$$=\frac{5-3x-(5+3x)}{(5+3x)(5-3x)}$$
$$=\frac{-6x}{25-9x^2}$$
$$=\frac{6x}{9x^2-25}$$

4. [B]

$m^2-2m-9n^2+6n$

$=m^2-9n^2-2m+6n$

$=(m+3n)(m-3n)-2(m-3n)$

$=(m-3n)(m+3n-2)$

5. [C]

$\mathrm{f}(x)=3x^2+x+2k$

$\mathrm{f}(k+1)-\mathrm{f}(k-1)$

$=[3(k+1)^2+(k+1)+2k]-[3(k-1)^2+(k-1)+2k]$

$=3(k+1)^2-3(k-1)^2+2$

$=3k^2+6k+3-3k^2+6k-3+2$

$=12k+2$

6. [A]

$\mathrm{g}(-x)=x^2-ax+b$

$\because\quad \mathrm{g}(x)=\mathrm{g}(-x)$

$\therefore\quad x^2+ax+b=x^2-ax+b$

$2ax=0$

$a=0$

$\mathrm{g}(x)=x^2+b$

$\mathrm{g}(-1)=1+b=-3$

$b=-4$

$\mathrm{g}(x)=x^2-4$

The remainder $=\mathrm{g}(-2)=4-4=0$

7. [A]

$x^2+(a+b)x\equiv(x+2)(x-3)+b$

$x^2+(a+b)x\equiv x^2-x-6+b$

$\therefore\quad a+b=-1$ and $-6+b=0$

Solving $b=6$, $a=-7$

Alternative solution:

$x^2+(a+b)x\equiv(x+2)(x-3)+b$

Sub. $x=0$, we have $-6+b=0$, $b=6$

$x^2+(a+b)x\equiv(x+2)(x-3)+6$

Sub. $x=3$, we have $9+3(a+6)=6$, $a=-7$

8. [D]

$y=-(px+3)^2+q$

$=-p^2(x+\dfrac{3}{p})+q$

The vertex is $(-\dfrac{3}{p}\,,\,q)$

Since the vertex lies in the quadrant II ,

so $p>0$ and $q>0$.

9. [D]

The cost $=160\times 85\%\div(1+8.8\%)$

$=125$

The percentage profit $=\dfrac{160-125}{125}\times 100\%$

$=28\%$

10. [B]

The actual area $=4\times 25000^2\text{ cm}^2$

$=4\times 250^2\text{ m}^2$

$=2.5\times 10^5\text{ m}^2$

11. [C]

$t=\dfrac{kp}{\sqrt{q}}$, k is a constant

$p_1=0.65p$, $q_1=1.69q$

$t_1=\dfrac{k(0.65p)}{\sqrt{1.69q}}$

$=0.5t$

$\therefore$ t is decreased by 50% .

12. [C]

$-5 < 3-2x < x+6$

From $-5 < 3-2x$, we have

$-8 < -2x$

$\therefore \quad x < 4$

From $3-2x < x+6$, we have

$-3x < 3$

$\therefore \quad x > -1$

$\therefore \quad -1 < x < 4$

13. [A]

Let $a_1 = a$. Then from $a_3 = 11$ and $a_{n+2} = 2a_n + a_{n+1}$,

we have $a_2 = 11-2a$, $a_4 = 33-4a$, $a_5 = 55-4a$,

and $a_6 = 121-12a$

$\therefore \quad 121-12a = 85$

$12a = 36$

$a = 3$

14. [D]

CB produced meets FE at N .

The area of the pentagon = The area of $CDEN$ + The area of $ABNF$

$\therefore \quad 3.5\times6.5+(9.5-3.5)\times3.5 \le y < 4.5\times7.5+(10.5-4.5)\times4.5$

i.e. $43.75 \le y < 60.75$

15. [C]

Let r and θ be the original radius and the angle at the centre of the sector respectively .

Then the new radius and the angle at the centre are $\frac{5}{4}r$ and $(1-k\%)\theta$ respectively .

$$\therefore \quad \pi r^2 \times \frac{\theta}{360°} = \pi(\frac{5}{4}r)^2 \times \frac{(1-k\%)\theta}{360°}$$

$$1 = \frac{25}{16}(1-k\%)$$

$$1-k\% = \frac{16}{25}$$

$$k = 36$$

16. [D]

The slant height of the triangle with the base of 10 is $\sqrt{12^2+16^2}=20$;

the slant height of the triangle with the base of 32 is $\sqrt{12^2+5^2}=13$.

The total surface area $=(\frac{10\times 20}{2}+\frac{32\times 13}{2})\times 2+32\times 10$

$=936\ (\text{cm}^2)$

17. [C]

$DE:EC=7:9$

Let $x\ \text{cm}^2$ be the area of ΔDEF ,

then $\frac{x}{x+32}=\frac{7^2}{9^2}$

$81x=49(x+32)$

$32x=49\times 32$

$x=49$

Note that $AB:DE=AF:FE=2:7$

The area of ΔAFB $=49\times\frac{2^2}{7^2}$,

$=4$

The area of ΔDAF $=49\times\frac{2}{7}$,

$=14$

The area of $ABCD$ $=32+49+4+14$,

$=99\ (\text{cm}^2)$

18. [A]

$\because$ $AB=BC=2CD$

$\therefore$ $AB:BC:CD=2:2:1$

$\because$ $\Delta ABE\sim\Delta ACF$

$\therefore$ $BE:CF=1:2$

$\because$ $\Delta DCG\sim\Delta DBE$

$\therefore$ $CG:BE=1:3$

and $CG:BE:GF=1:3:5$

Trapezium $BCGE$ and ΔEFG have the same height ,

the ratio of the two areas $=\frac{1+3}{2}:\frac{5}{2}$

$=4:5$

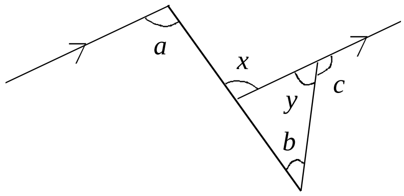
19. [A]

$x = a$

$y = x - b = a - b$

and $y + c = 180°$

i.e. $a - b + c = 180°$ (I is true)



If II. is true ,

then $2a = 360°$, $a = 180°$

Thus, II. is false ,

If III. is true, then $2b = 90°$, $b = 45°$

Thus, III. must not be true .

20. [A]

$\because$ $BC = BE$

$\therefore$ $\angle CBE = 180° - 2 \times 56° = 68°$

$\because$ $AB = BC = BE$

$\therefore$ $\angle BAE = \dfrac{180° - (90° + 68°)}{2} = 11°$

$\angle AFD = 11° + 45° = 56°$

21. [B]

Join BD .

$BD = \sqrt{9^2 + 12^2} = 15$

$\because$ $BC^2 + BD^2 = 64 + 225 = 289 = CD^2$

$\therefore$ $\angle CBD = 90°$

$\tan \angle ADB = \dfrac{9}{12}$, $\tan \angle DBC = \dfrac{8}{15}$

$\therefore$ $\angle ADB = 36.870°$, $\angle DBC = 28.072°$

$\angle ADC = \angle ADB + \angle BDC$

$= 36.870° + 28.072°$

$= 64.942°$

$= 65°$

22. [C]

Join CA and AE .

$\because \angle ABC = 90°$

$\therefore$ CA is the diameter of the circle .

$\angle AEC = 90°$

$\angle DCA = 180° - (90° + 36°) = 54°$

Let X be the centre ,

then $XC = XD = \dfrac{3}{\cos 54°}$

The area of the circle $= \pi \times (\dfrac{3}{\cos 54°})^2 = 81.838 = 82 \ (\text{cm}^2)$

23. [B]

$\angle DEC = \beta$

In ΔDEC , $EC = \dfrac{DC}{\tan \beta}$

In ΔABC , $AC = \dfrac{AB}{\cos \alpha} = \dfrac{DC}{\cos \alpha}$

$\therefore \dfrac{EC}{AC} = \dfrac{\cos \alpha}{\tan \beta}$

24. [C]

Note that POQ is a straight line , where O is the pole .

$\angle ROQ = 350° - 290° = 60°$

The area of ΔPQR $= \dfrac{1}{2} \times (5+3) \times 6 \sin 60°$

$= 12\sqrt{3}$

25. [D]

Obviously, I. is true.

Consider $\dfrac{x}{b} + \dfrac{y}{c} = 1$. Let $x = 0$, we have $y = c$; let $y = 0$, we have $x = b$

$\therefore$ b and c are negative numbers .

Thus, II. is true .

From $a < 0$ and $b < 0$, we have $ab > 0$

and $b > \dfrac{1}{a}$, so $ab < 1$ ($\because$ $a < 0$)

$\therefore$ III. is also true .

26. [B]

Let h he the height of ΔPAB .

Then $\frac{1}{2}(AB)(h) = 20$

Since AB is a constant , h is also a constant .

$\therefore$ The locus of P is a pair of parallel lines .

27. [A]

$x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$

$(x-h)^2 + (y-k)^2 = r^2$

Centre is (h, k) , radius is r $(r > 0)$.

From the graph , we know that the centre lies in the quadrant III ,

so $h < 0$ and $k < 0$.

We also know from the graph that $-h > r$ and $r > -k$

So $r + h < 0$ and $r + k > 0$

i.e. I. and II. are true ,

and $-h > -k$

$k - h > 0$

i.e. III. is false .

28. [D]

The required probability $= \frac{14}{20} = \frac{7}{10}$

29. [B]

Obviously I. and III. are true , but II. is false .

30. [D]

When $h = 9$, A. is false .

When $h = 6$, B. is false .

When $h = 6$, C. is false .

When $h = 9$, the data has the greatest inter-quartile range ,

its value $= 7.5 - 4 = 3.5 < 4$

$\therefore$ D. must be true .

31. [C]

$$8^{17}+8^4-8^3$$
$$=(2^3)^{17}+8^3(8-1)$$
$$=2^{51}+7(2^3)^3$$
$$=2^{51}+7(2^9)$$
$$=2^{48}\cdot 2^3+7(2^8\cdot 2)$$
$$=(2^4)^{12}\cdot 8+14(2^4)^2$$
$$=8\times 16^{12}+14\times 16^2$$
$$=8000000000E00_{16}$$

32. [A]

When the graph on the right is $y=\mathrm{f}\,(x)$, then the graph on the left is $y=-\mathrm{f}\,(-x)$.

33. [D]

$$(\log_a x)^2+4\log_a x^2-18=\log_a x$$
$$(\log_a x)^2+8\log_a x-18=\log_a x$$
$$(\log_a x)^2+7\log_a x-18=0$$
$$(\log_a x+9)(\log_a x-2)=0$$
$$\log_a x=-9 \quad \text{or} \quad \log_a x=2$$
$$x=\frac{1}{a^9} \quad \text{or} \quad x=a^2$$

Product of roots $mn=\frac{1}{a^9}\cdot a^2=\frac{1}{a^7}$

34. [B]

I. is true , but II. is false .

$AC=-\log_a y$, $BC=-\log_b y$

$$\frac{AB}{BC}=\frac{AC-BC}{BC}$$
$$=\frac{AC}{BC}-1$$
$$=\frac{-\log_a y}{-\log_b y}-1$$
$$=\frac{\log_a y}{\frac{\log_a y}{\log_a b}}-1$$
$$=\log_a b-\log_a a$$
$$=\log_a\frac{b}{a}$$

35. [B]

Obviously , I. is an arithmetic sequence , while II. is not an arithmetic sequence (It is a geometric sequence) .

$\because$ $7\log\sqrt{a}-3\log\sqrt{a}=4\log\sqrt{a}$

$$3\log\sqrt{a}-\log\frac{1}{\sqrt{a}}=3\log\sqrt{a}-\log(\sqrt{a})^{-1}$$
$$=3\log\sqrt{a}+\log\sqrt{a}$$
$$=4\log\sqrt{a}$$

$\therefore$ III. is an arithmetic sequence .

36. [A]

$(k-2i)(2+ki)^2$

$=(k-2i)(4-k^2+4ki)$

$=k(4-k^2)+8k-8i+2k^2i+4k^2i$

Real part $=k(4-k^2)+8k$

$=-k^3+12k$

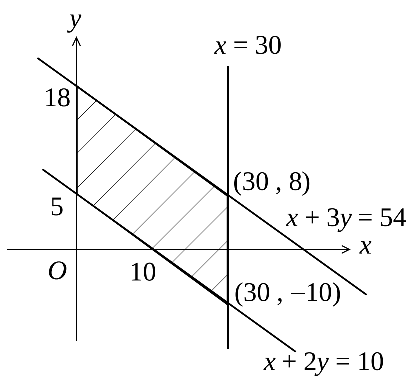
37. [C]

Let $f(x, y)=2x-3y+2$

$f(0, 5)=-13$, $f(0, 18)=-52$

$f(30, 8)=38$, $f(30, -10)=92$

The greatest value of $f(x, y)$ is 92 .



38. [D]

Construct the vertical line XN such that it meets the plane $EFGH$ at N , and join FN .

Then $FN=\sqrt{b^2+c^2}$, $XN=2a$

and $XF=\sqrt{4a^2+b^2+c^2}$

Construct $XM\perp AD$ such that the foot of the perpendicular is M , and join MF .

$$MF=\sqrt{4a^2+c^2}$$

$$\theta=\angle XFM$$

$$\therefore\quad \cos\theta=\frac{MF}{XF}=\frac{\sqrt{4a^2+c^2}}{\sqrt{4a^2+b^2+c^2}}$$

39. [A]

Join DC .

$\angle DCE = \angle BDE = 35°$

$\angle CDE = \angle BCQ = 65°$

$\angle DEC = 180° - 35° - 65° = 80°$

$\angle DBE = 80° - 35° = 45°$

$\angle BFC = \angle BCQ - \angle DBE$

$= 65° - 45°$

$= 20°$

40. [D]

$4x + 3y + k = 0$

$y = -\dfrac{4x+k}{3}$

Sub. into $x^2 + y^2 + 2x - 2y - 2 = 0$

We have $x^2 + (-\dfrac{4x+k}{3})^2 + 2x - 2(-\dfrac{4x+k}{3}) - 2 = 0$

$9x^2 + (4x+k)^2 + 18x + 6(4x+k) - 18 = 0$

$25x^2 + (8k+42)x + k^2 + 6k - 18 = 0$

$\Delta = (8k+42)^2 - 100(k^2 + 6k - 18) < 0$

$-36k^2 + 72k + 3564 < 0$

$k^2 - 2k - 99 > 0$

$(k-11)(k+9) > 0$

$\therefore \quad k < -9$ or $k > 11$

Alternative solution:

$x^2 + y^2 + 2x - 2y - 2 = 0$

Centre $= (-1, 1)$, radius $= \dfrac{1}{2}\sqrt{4+4+8} = 2$

The perpendicular distance between the centre and the straight line $4x + 3y + k = 0$

$= \left|\dfrac{4(-1)+3(1)+k}{\sqrt{4^2+3^2}}\right| = \left|\dfrac{k-1}{5}\right|$

$\because$ The circle and the straight line do not intersect ,

$\therefore \quad \left|\dfrac{k-1}{5}\right| > 2$

$\dfrac{k-1}{5} < -2$ or $\dfrac{k-1}{5} > 2$

$\therefore \quad k < -9$ or $k > 11$

41. [B]

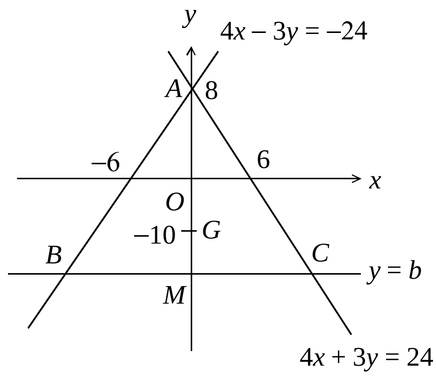
Obviously, ABC is an isosceles triangle

Centroid G lies on the y-axis .

$\because \quad AG = 18$

$\therefore \quad GM = 9$

$\therefore \quad b = -10 - 9 = -19$



42. [B]

The required permutations $= C_2^4 \times P_3^6 \times P_2^2 \times P_5^5$

$= 172800$

43. [C]

The required probability $= \frac{2}{3} \times \frac{2}{3}$

$= \frac{4}{9}$

44. [A]

Let x_1 and x_2 be the test scores of two students and $x_1 > x_2$.

And let m be the mean score of the test .

Then the difference of the standard scores of the two students

$= \frac{x_1 - m}{6} - \frac{x_2 - m}{6}$

$= \frac{x_1 - x_2}{6}$

$= \frac{18}{6}$

$= 3$

45. [D]

The standard deviation of the last 5 data before amendment $=$ The standard deviation of the first 5 data .

$\therefore$ The variance of the last 5 data after amendment

$= 2^2 \times 2^2$

$= 16$